Surname: . . . . **First Name:** . . . . **Student ID:** . . . .

VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

CLASS TEST 4 – 2021

TRIMESTER 2

EEEN 220

SIGNALS, SYSTEMS AND STATISTICS 1

Time Allowed: FIFTY MINUTES

CLOSED BOOK

Permitted materials: Calculators.
Statistical tables
One double-sided A4 paper of notes

Instructions: Attempt ALL Questions.
Only partial marks are awarded for correct answers if working/reasoning is not shown.
NOT all questions have the same marks value.
There are 42 marks in total.

1. Exponential and Poisson Distributions **(12 marks)**

   The distance between flaws on a long cable is exponentially distributed with mean 7 metres.

   (a) Find the probability that the distance between two flaws is greater than 13 metres.

   [2 marks]

   (b) Find the probability that a 10 length of cable contains exactly 2 flaws.

   [4 marks]

   (c) Find the median distance between flaws.

   [4 marks]

(d) Find the standard deviation of the distances

[2 marks]

2. Queueing Theory **(10 marks)**

The Inland Revenue Department (IRD) has 100 agents at a call centre responding to calls to a particular 0800 number. They estimate that during the busy hour the (Poisson) call arrival rate is 6 calls per minute. The mean call duration (assumed exponentially distributed) is 14 minutes.

(a) What is the busy hour traffic load? (provide the value and the units)
[2 marks]

(b) If we assume that callers will wait for an agent indefinitely, what is the approximate probability that a caller will need to wait?
[4 marks]

(c) If we instead assume that callers not served immediately will not wait at all, but will simply hang up, and not call again in the busy hour, what is the approximate probability that a caller will immediately hang up?

[4 marks]

3. Maximum likelihood estimation **(8 marks)**

Consider the continuous uniform distribution, with pdf given by

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

An iid sample $\{x_1\, x_2, \ldots, x_N\}$ of size N is drawn from this distribution.

Show that the maximum likelihood estimators of parameters a and b are given by

$$\hat{a} = \min_i x_i$$
$$\hat{b} = \max_i x_i$$

(Hint: calculus is not required, but some careful thought about the maximum likelihood estimator, and a clear explanation of your thinking *are* required.)

4. Central Limit Theorem **(12 marks)**

The concentration of particles in a suspension is 50 per mL. A 8 mL volume of the suspension is withdrawn.

(a) What is the probability that the number of particles withdrawn will be between 381 and 419?

[3 marks]

(b) What is the probability that the *average* number of particles per mL in the withdrawn sample is between 46 and 54?

[4 marks]

(c) How large a sample must be withdrawn so that the average number of particles per mL in the sample is between 46 and 54 with probability 82%?

[5 marks]

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