

Surname:

First Name:

Student ID:



TEST 1 – 2019

TRIMESTER 1

ECEN 203
ANALOGUE CIRCUITS
AND SYSTEMS

Time Allowed: FIFTY MINUTES

CLOSED BOOK

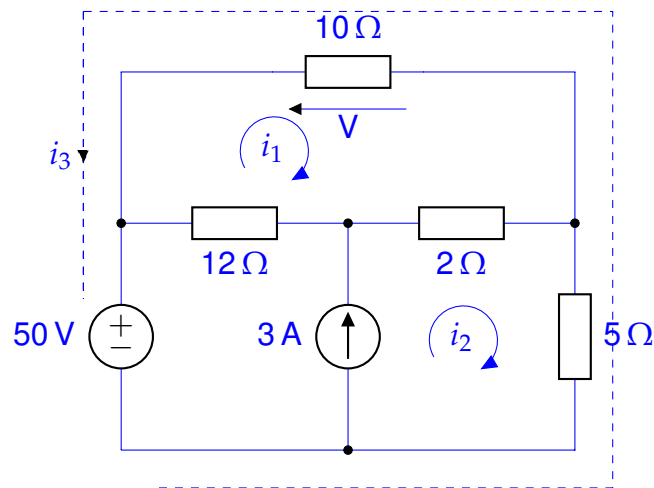
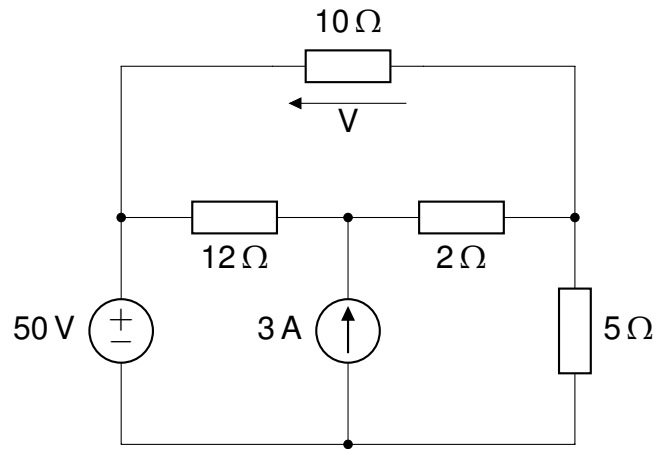
Permitted materials: You may use a scientific calculator, but not a smartphone or other similar device.

Instructions: Attempt ALL THREE Questions
Write your answers on the question paper.
Full marks will be awarded for correct answers.
Partial marks may be awarded to incorrect answers if working is shown.
The exam will be marked out of a total of 50 marks.

1. Mesh current analysis

(20 marks)

Find V by mesh current analysis. Note: you will need to come up with a strategy to deal with the current source.



Remember that the key is to make sure that only one current flows through the current source, hence the need for the loop around the outside, or perhaps around the bottom half.

(4 marks)

$$\begin{aligned}
 -10i_1 - 2i_1 - 12i_1 + 2i_2 + 10i_3 &= 0 \\
 i_2 &= 3 \\
 10i_1 + 5i_2 - 5i_3 - 10i_3 - 50 &= 0
 \end{aligned}$$

(6 marks)

$$\begin{aligned}
 -24i_1 + 10i_3 &= -6 \\
 10i_1 - 15i_3 &= 35
 \end{aligned}$$

$$12i_1 - 5i_3 = 3$$

$$2i_1 - 3i_3 = 7$$

$$\begin{bmatrix} 12 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

(2 marks)

$$\begin{vmatrix} 12 & -5 \\ 2 & -3 \end{vmatrix} = (12)(-3) - (2)(-5) = -36 + 10 = -26$$
$$\begin{vmatrix} 3 & -5 \\ 7 & -3 \end{vmatrix} = (3)(-3) - (7)(-5) = -9 + 35 = 26$$
$$\begin{vmatrix} 12 & 3 \\ 2 & 7 \end{vmatrix} = (12)(7) - (2)(3) = 84 - 6 = 78$$

(3 marks)

So

$$i_1 = \frac{26}{-26} = 1 \text{ A}$$

$$i_3 = \frac{78}{-26} = -3 \text{ A}$$

(2 marks)

So the current through the 10Ω resistor is $i_1 - i_3 = -1 - -3 = 2 \text{ A}$ to the right.

(2 marks)

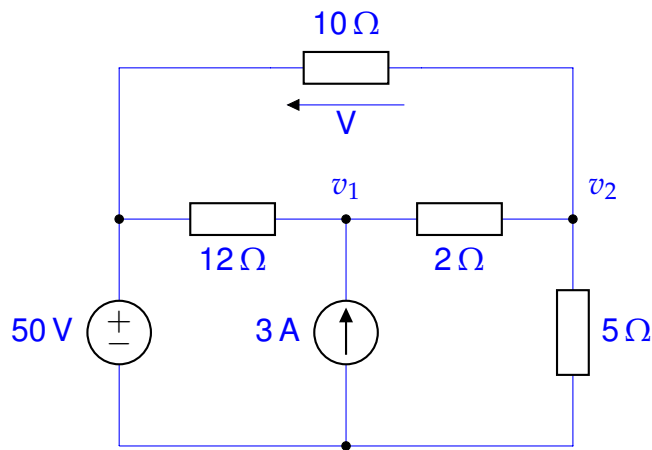
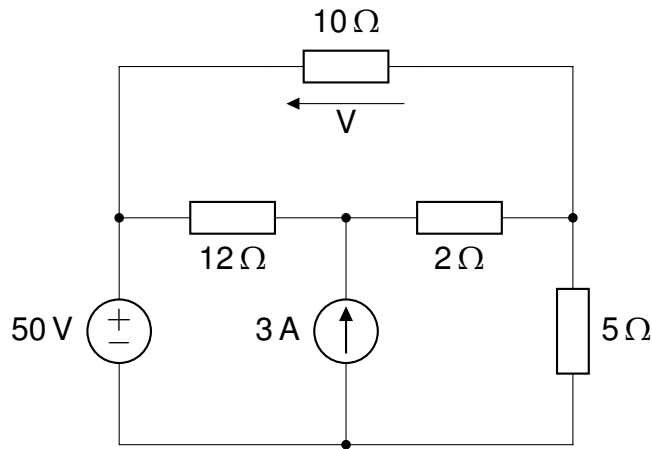
$$\text{So } V = (10)(2) = +20 \text{ V}$$

(1 mark)

2. Node voltage analysis

(15 marks)

Find V by node voltage analysis. (Note: this is the same circuit as the previous question).



(2 marks)

At node 1:

$$\frac{v_1 - 50}{12} - 3 + \frac{v_1 - v_2}{2} = 0$$

(3 marks)

At node 2:

$$\frac{v_2 - v_1}{2} + \frac{v_2}{5} + \frac{v_2 - 50}{10} = 0$$

(3 marks)

$$\begin{bmatrix} \frac{7}{12} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{8}{10} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{86}{12} \\ \frac{5}{5} \end{bmatrix}$$

(2 marks)

$$\begin{bmatrix} 7 & -6 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 86 \\ 50 \end{bmatrix}$$

Note that we don't need to solve this for v_1 , just v_2 :

$$\begin{aligned} v_2 &= \frac{\begin{vmatrix} 7 & 86 \\ -5 & 50 \end{vmatrix}}{\begin{vmatrix} 7 & -6 \\ -5 & 8 \end{vmatrix}} = \frac{350 + 430}{56 - 30} \\ &= \frac{780}{26} \\ &= 30 \end{aligned}$$

(3 marks)

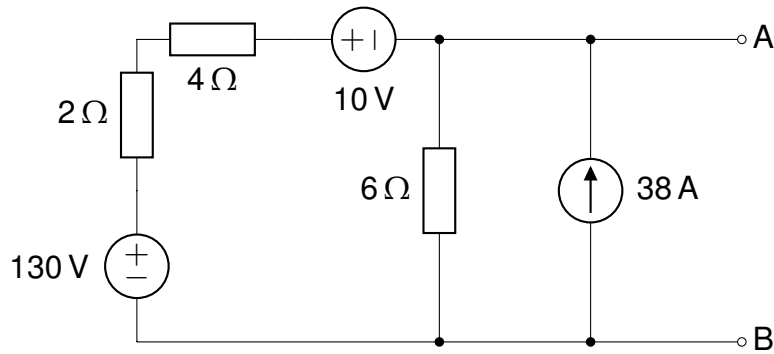
$$\text{So } v = 50 - 30 = 20 \text{ V}$$

(2 marks)

3. Thevenin/Norton equivalents

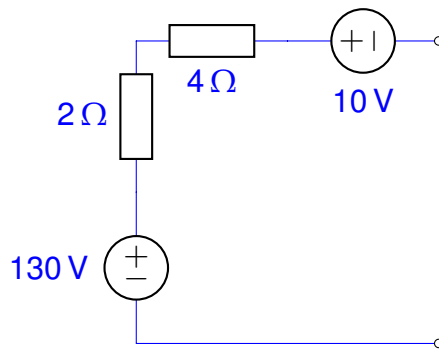
(15 marks)

Find the Thevenin equivalent at terminals A&B:

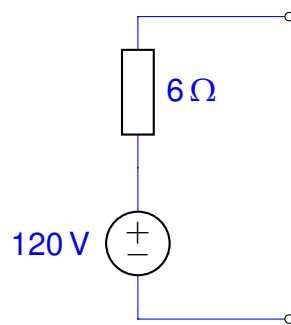


There are many ways to do this.

One of the easiest is to replace:

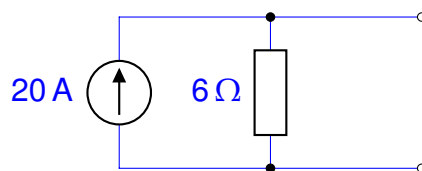


by



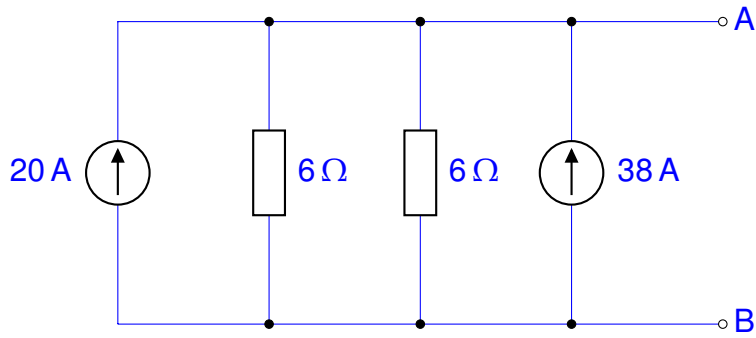
(5 marks)

and then by its Norton equivalent



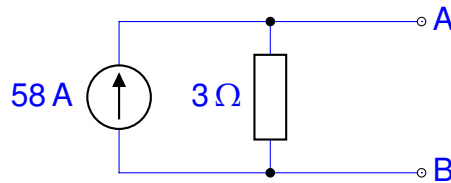
(3 marks)

And then we have



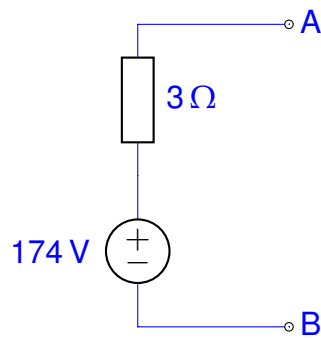
(2 marks)

and hence



(3 marks)

and so the Thevenin equivalent is



(2 marks)

It is actually very easy to get the Thevenin resistance, by zeroing the sources to get $6 \parallel 6 = 3 \Omega$, but finding the open-circuit voltage or the short-circuit current is possibly a bit harder than the transformations above.

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TEST 2 – 2019

TRIMESTER 1

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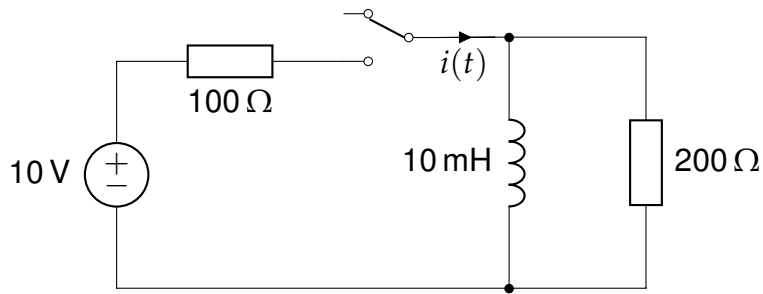
CLOSED BOOK

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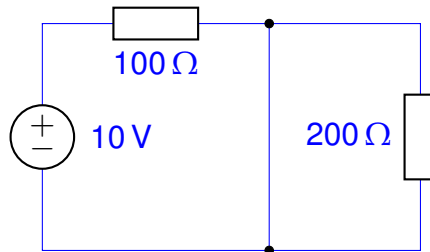
1. Transient analysis

(20 marks)



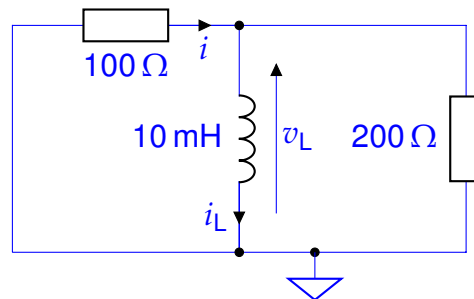
The switch closes at $t = 0$ after being open for a long time. Calculate an expression for and plot $i(t)$. (It may be easiest to separately calculate the steady state response and the transient response).

For the steady-state response $\frac{di}{dt} = 0$ so $v_L = 0$, so in effect we have



So $i_{ss}(t) = \frac{10}{100} = 0.1 \text{ A}$

For the transient response, (i.e., with the source removed) we have:



Using node voltage analysis:

$$i = -\frac{v_L}{100}$$

and

$$\frac{v_L}{100} + i_L + \frac{v_L}{200} = 0$$

$$\frac{3v_L}{200} + i_L = 0$$

$$v_L = L \frac{di_L}{dt} = 10^{-2} \frac{di_L}{dt} - \frac{200}{3} i_L$$

$$\frac{di_L}{dt} = -\frac{20000}{3} i_L$$

$$i_L = A e^{-\frac{20000}{3}t} = A e^{-\frac{t}{150 \mu\text{s}}}$$

(We could also have got this from considering the Thevenin equivalent of the voltage source and resistors.)

$$\begin{aligned} i_{\text{tr}}(t) &= i_L(t) + \frac{v_L}{200} \\ &= i_L(t) + \frac{1}{200} L \frac{di_L}{dt} \\ &= A e^{-\frac{t}{150 \mu\text{s}}} + \frac{1}{200} 10 \times 10^{-3} \left(\frac{-1}{150 \mu\text{s}} \right) e^{-\frac{t}{150 \mu\text{s}}} \\ &= B e^{-\frac{t}{150 \mu\text{s}}} \end{aligned}$$

i.e., the transient response of $i(t)$ takes the same form as $i_L(t)$, and we don't actually need to find out what A is.

At the moment the switch closes, $i_L(0^+) = i_L(0^-) = 0$, so $i(0) = \frac{10}{30} = \frac{1}{30}$

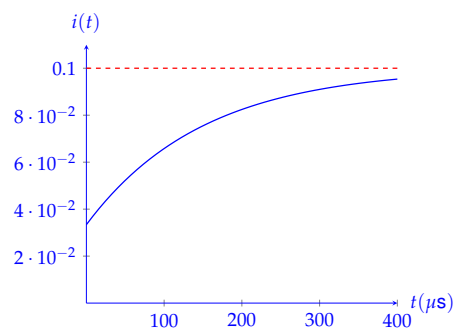
So the total response is

$$\begin{aligned} i_{\text{total}} &= i_{\text{ss}} + i_{\text{tr}} \\ &= 0.1 + B e^{-\frac{t}{150 \mu\text{s}}} \\ i_{L,\text{total}}(0) &= \frac{1}{30} = \frac{1}{10} + B \end{aligned}$$

so $B = -\frac{2}{30}$

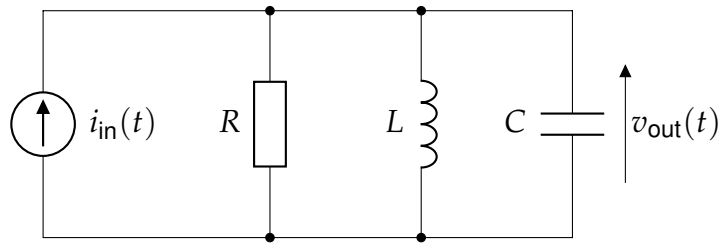
so

$$i(t) = \frac{1}{30} \left(3 - 2e^{-\frac{t}{150 \mu\text{s}}} \right)$$



2. LCR circuits and frequency response

(30 marks)



- (a) **(10 marks)** (Phasors) For the circuit above, write a general expression for the output voltage phasor V_{out} in terms of the input current phasor I_{in} .

The output voltage is the input current times the combined impedance of the three parallel components:

$$V_{out} = I_{in} \left(\frac{1}{sC + \frac{1}{R} + \frac{1}{sL}} \right)$$

$$\frac{V_{out}(s)}{I_{in}(s)} = \frac{sRL}{s^2RLC + sL + R}$$

Since we are next told that the input is a sinusoid of constant magnitude, $s = j\omega$, and the above is equivalent to

$$\frac{V_{out}(\omega)}{I_{in}(\omega)} = \frac{j\omega RL}{-\omega^2RLC + j\omega L + R}$$

- (b) **(10 marks)** (Impedance) If $R = 50\ \Omega$, $L = 0.1\ \text{H}$, $C = 10\ \mu\text{F}$, and i_{in} is a sinusoid of rms value $10\ \text{A}$ and frequency $63.662\ \text{Hz}$, calculate the amplitude and phase of the output voltage v_{out} and its phase relative to i_{in} . Hence write an expression for v_{out} .

$$\omega = 2\pi f = 2\pi 63.662 = 400.0001 \approx 400\ \text{rad s}^{-1}$$

Also the peak amplitude is given by

$$10\sqrt{2} = 14.142\ \text{A}$$

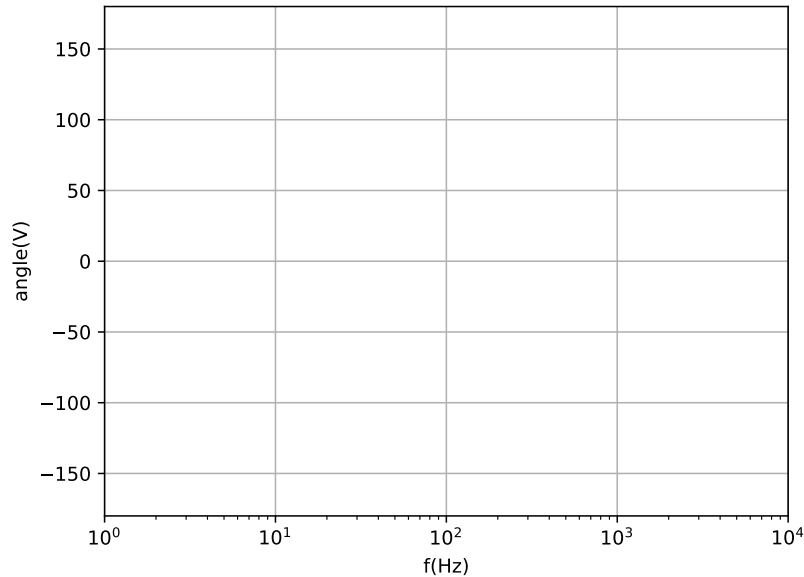
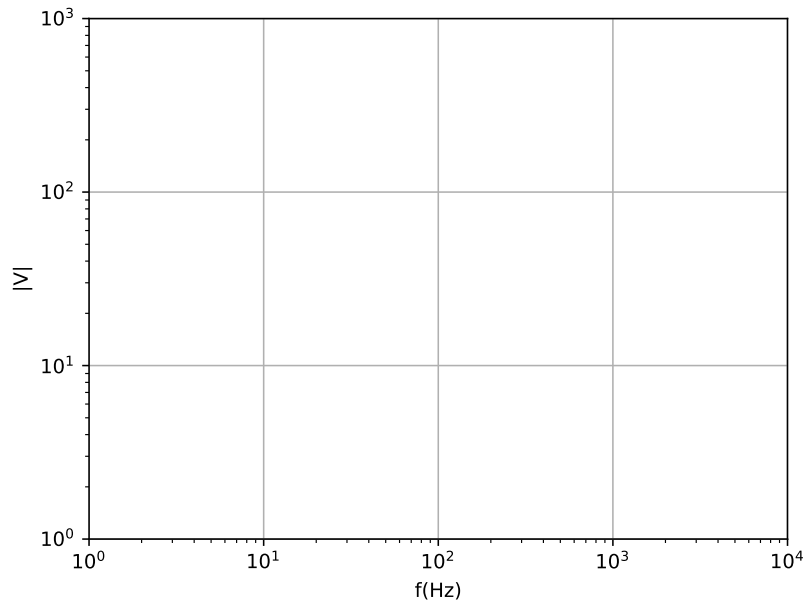
So we have

$$\begin{aligned} V_{out} &= \frac{j(50)(400)(0.1)}{-400^2(50)(0.1)(10^{-5}) + 50 + j(400)(0.1)} 10\sqrt{2} \\ &= \frac{j2000}{50 - 8 + j40} 10\sqrt{2} \\ &= \frac{j2000}{42 + j40} 10\sqrt{2} \\ &= \frac{j2000(21 - 20j)}{2 \times 29^2} 10\sqrt{2} \\ &= \frac{1000(20 + 21j)}{29^2} 10\sqrt{2} \\ &= (23.781 + 24.970j) 14.142 \\ &= 336.3 + 353.1j \\ &= 487.7 \angle 0.8097 \\ &= 487.7 \angle 46.4^\circ \end{aligned}$$

So $v_{out}(t) = 487.7 \cos(400t + 0.8097)\ \text{V}$

(c) **(10 marks)** (Frequency response) If the frequency of the source retains the same amplitude, but varies in frequency between 1 Hz and 10 kHz, plot the output voltage V_{out} amplitude and phase as a function of frequency on the graphs below.

The plots do not need to be particularly accurate, but should include certain features that we would expect.

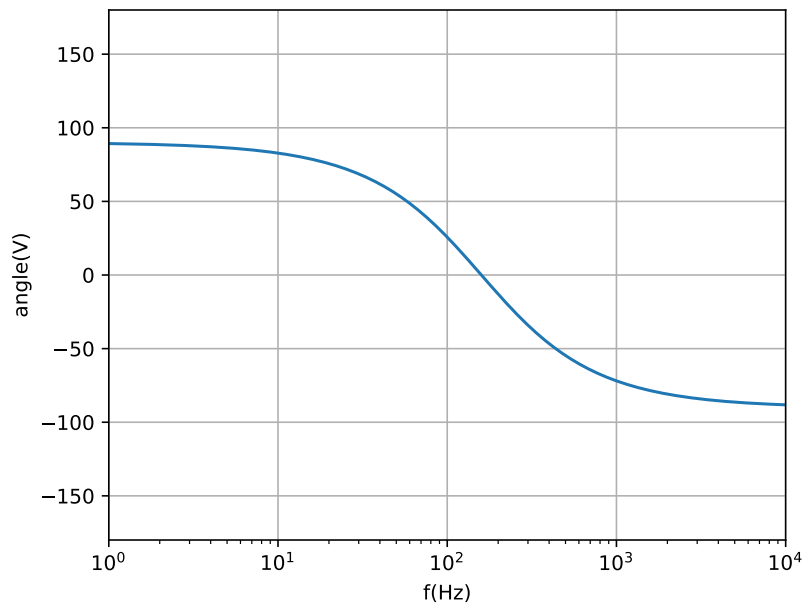
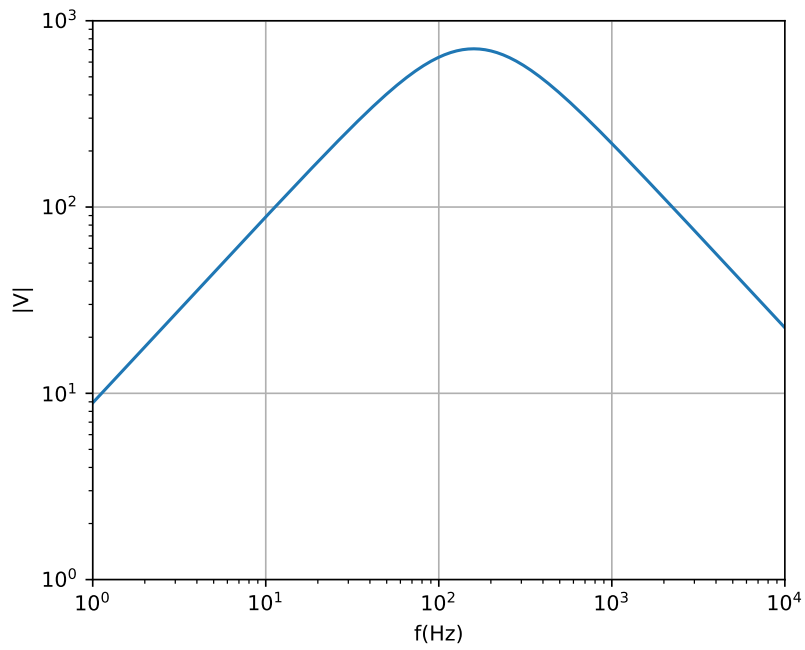


At very low frequencies, the inductor is a short circuit, while the capacitor is an open circuit. So we expect the circuit to behave like a high pass filter with a 20 dB/decade slope, and phase approaching $+90^\circ$.

At very high frequencies, the inductor is an open circuit, while the capacitor is a short circuit. So we expect the circuit to behave like a low pass filter with a 20 dB/decade slope, and phase approaching -90° .

At $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^6}} \approx 159$ Hz, the real part of the denominator is zero,

so the phase is zero, and the output reaches its maximum value of 707.1 V.



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TEST 3 – 2019

TRIMESTER 1

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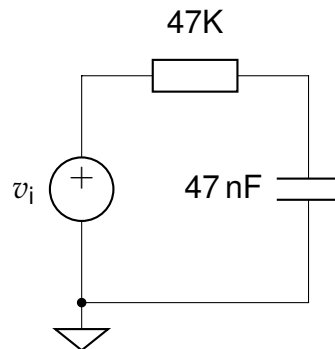
CLOSED BOOK

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Instructions: Attempt ALL Questions
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1. Frequency Response

(10 marks)



- (a) **(5 marks)** $v_i(t)$ is a 80 Hz cosinusoid having amplitude of 5 V. What steady state current will flow through the resistor? Write your answer as a function of time.

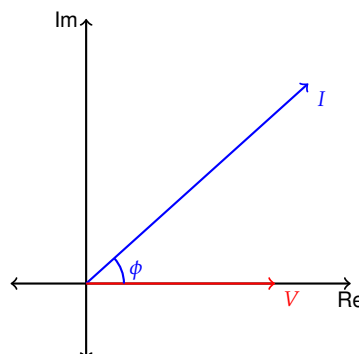
$$\begin{aligned}
 I(\omega) &= \frac{V_i(\omega)}{R + \frac{1}{j\omega}} \\
 &= \frac{5}{47 \times 10^3 + \frac{1}{j2\pi 80 \times 47 \times 10^{-9}}} \\
 &= \frac{5}{47000 - 42328.44j} \\
 &= \frac{5}{63.251 \times 10^3 \angle -42.0^\circ} \\
 &= (5.874 + 5.290j) \times 10^{-5} \\
 &= 7.905 \times 10^{-5} \angle 42.0^\circ
 \end{aligned}$$

so

$$i(t) = 79.05 \cos(160\pi t + 42.0^\circ) \mu\text{A}$$

- (b) **(5 marks)** Does the current lead or lag the input voltage of this circuit? Using a phasor diagram, explain why your answer is correct.

The current leads the voltage. Since both voltage and current can be considered to be rotating counter-clockwise, the phasor diagram shows that for any given angle, the current arrow “arrives first.”



2. Frequency response

(10 marks)

A circuit has transfer function $G(j\omega) = \frac{V_o}{V_i} = \frac{500}{j\omega+200}$. If an input voltage of $v_i(t) = 10 \cos(400t - 70^\circ)$ is applied to the circuit, what will the steady state output be? Express your answer as a function of time.

$$\omega = 400$$

So

$$\begin{aligned} G(j\omega) &= \frac{500}{400j + 200} \\ &= \frac{500}{447.214 \angle 1.107 \text{rad}} \\ &= \frac{500}{447.214 \angle 63.435^\circ} \\ &= 1.118 \angle -63.435^\circ \end{aligned}$$

$$V_i = 10 \angle -70^\circ$$

So

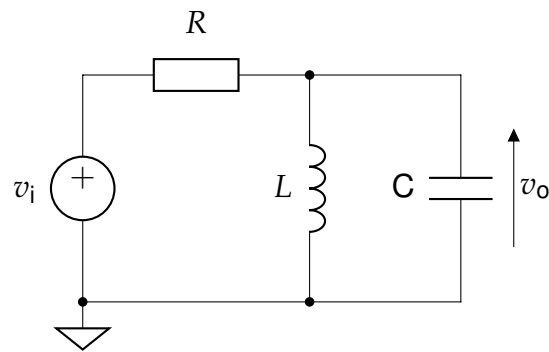
$$\begin{aligned} V_o &= (10 \angle -70^\circ)(1.118 \angle -63.435^\circ) \\ &= 11.18 \angle -133.435^\circ \end{aligned}$$

So

$$v_o(t) = 11.18 \cos(400t - 133.435^\circ) \text{ V}$$

3. Circuit transfer function

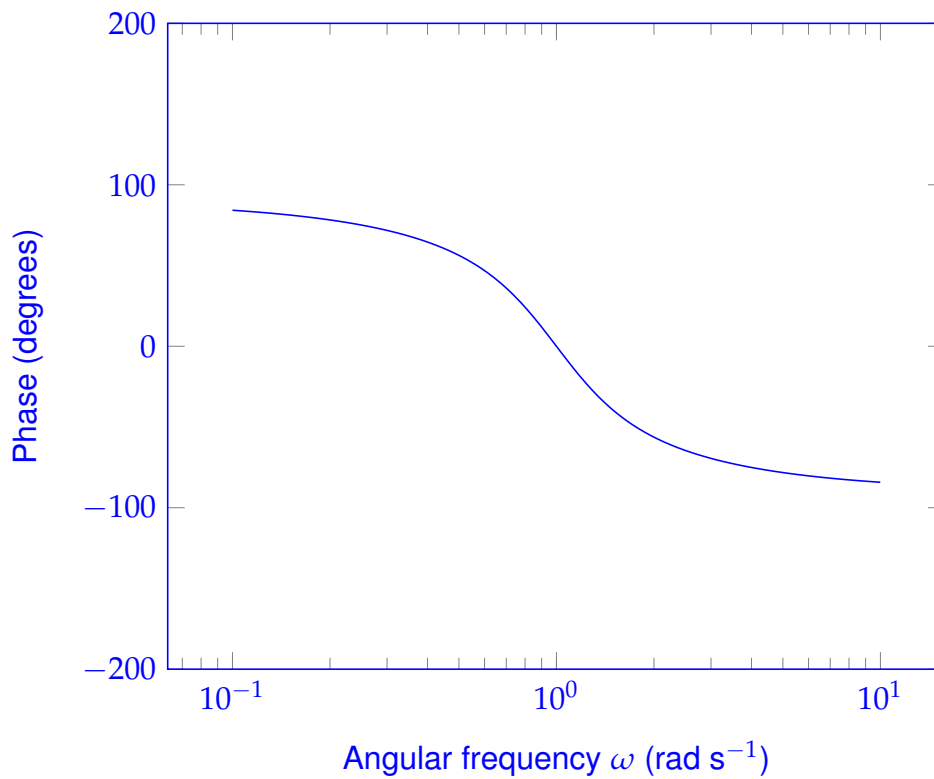
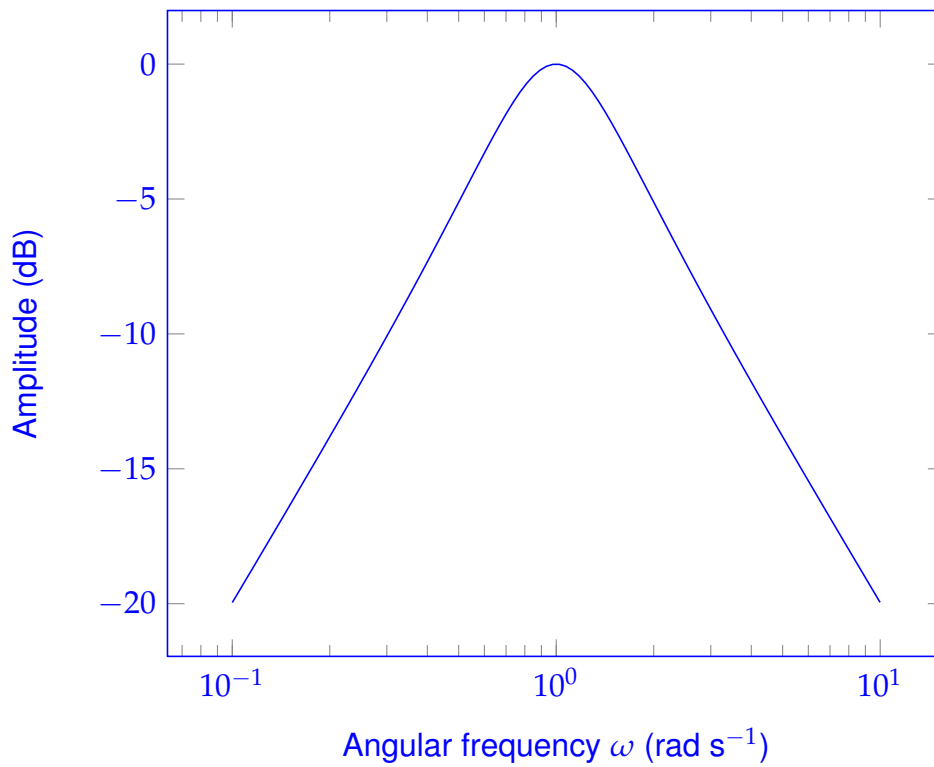
(10 marks)



- (a) (5 marks) What is the transfer function V_o/V_i of the circuit shown above? Write your answer so that the coefficient of the highest power of ω in the denominator is 1.

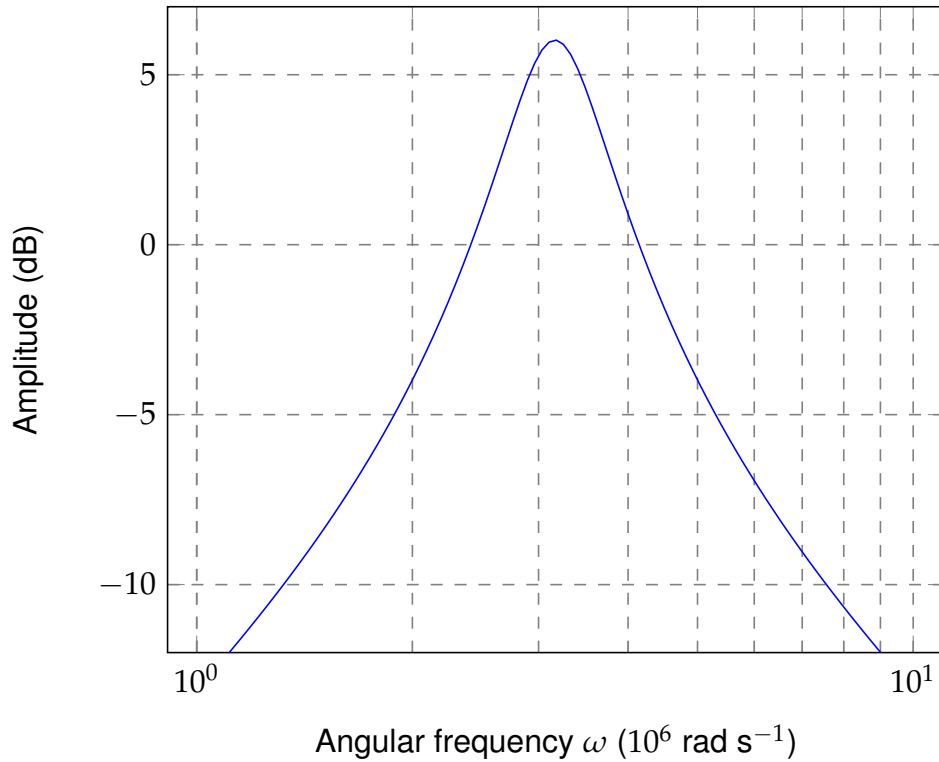
$$\begin{aligned}
 \frac{V_o(s)}{V_i(s)} &= \frac{Z_L \parallel Z_C}{R + Z_L \parallel Z_C} \\
 &= \frac{\frac{sL}{sC}}{sL + \frac{1}{sC}} \\
 &= \frac{\frac{sL}{sC}}{R + \frac{sL}{sL + \frac{1}{sC}}} \\
 &= \frac{\frac{sL}{s^2LC + 1}}{R + \frac{sL}{s^2LC + 1}} \\
 &= \frac{sL}{s^2LCR + sL + R} \\
 &= \frac{s \frac{1}{RC}}{s^2 + s \frac{1}{RC} + \frac{1}{LC}} \\
 \frac{V_o(j\omega)}{V_i(j\omega)} &= \frac{j\omega \frac{1}{RC}}{-\omega^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \\
 &= \frac{-j\omega \frac{1}{RC}}{\omega^2 - j\omega \frac{1}{RC} - \frac{1}{LC}}
 \end{aligned}$$

- (b) **(5 marks)** By considering the response at various frequencies (for $R = L = C = 1$ for example) sketch the form of the magnitude and phase responses of this circuit



4. Frequency response

(11 marks)



Consider the magnitude response of a resonant system shown above.

(a) **(2 marks)** What are the half power frequencies of the system?

The peak is 6 dB, so half power will be at 3 dB. These are at 2.7016×10^6 and $3.7016 \times 10^6 \text{ rad s}^{-1}$

(b) **(2 marks)** What is the bandwidth of the system?

So the bandwidth is $1 \times 10^6 \text{ rad s}^{-1}$

(c) **(2 marks)** What is the Q of the system?

The centre frequency is $\sqrt{10} \times 10^6 = 3.1623 \times 10^6 \text{ rad s}^{-1}$, so the Q is $3.1623/1 = 3.1623$

- (d) **(5 marks)** You are required to design a parallel resonant circuit having this magnitude response using a $10 \mu\text{H}$ inductor. What value resistor and capacitor would you select? (You are *not* restricted to use E12 values.)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

so

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{10 \times 10^{12} * 10 \times 10^{-6}} = 10^{-8}$$

so use a 10 nF capacitor.

For a parallel circuit, $Q = R\sqrt{\frac{C}{L}}$ so

$$R = Q\sqrt{\frac{L}{C}} = \sqrt{10}\sqrt{\frac{10^{-5}}{10^{-8}}} = 3162.3$$

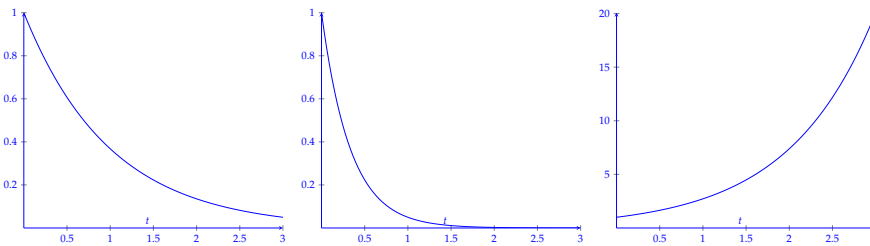
so use a 3.2 k Ω resistor.

5. Differential equation modes

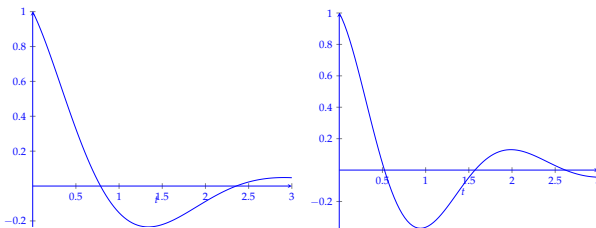
(9 marks)

The solutions to the constant coefficient linear differential equations that we encounter when dealing with circuit analysis have solutions that are made up of complex exponential terms $e^{(\sigma+j\omega)t}$. For each of the following, sketch the REAL PARTS of the modes (the waveforms) that correspond to the particular choice of $\sigma + j\omega$. (You don't need to be precise about this; the important thing is to distinguish between the different plots, not to plot them accurately).

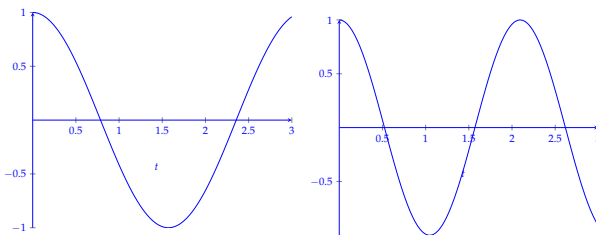
(a) (3 marks) $\sigma = -1, \sigma = -3$ and $\sigma = 1$, each with $\omega = 0$.



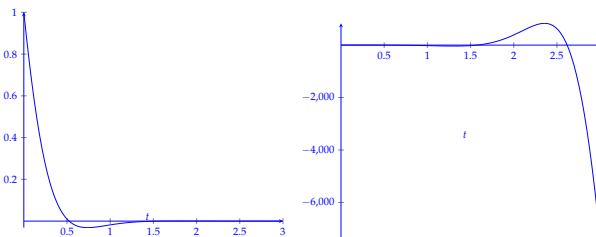
(b) (2 marks) $\omega = 2, \omega = 3$, both with $\sigma = -1$.



(c) (2 marks) $\omega = 1, \omega = 2$, both with $\sigma = 0$.



(d) (2 marks) $\sigma = -4$ and $\sigma = 3$, both with $\omega = 3$.



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TEST 4 – 2019

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1. Solving differential equations

(12 marks)

(a) (5 marks)

A system is governed by the differential equation

$$y''(t) + 5y'(t) + 4y(t) = 6x''(t) + 16x'(t) + 4x(t)$$

Assuming zero initial conditions, show that the transfer function of the system can be written as

$$G(s) := \frac{Y(s)}{X(s)} = \frac{6s^2 + 16s + 4}{s^2 + 5s + 4}$$

Laplace transform both sides of the equations, remembering that all initial conditions are zero.

$$s^2Y(s) + 5sY(s) + 4Y(s) = 6s^2X(s) + 16sX(s) + 4X(s)$$

$$\left[s^2 + 5s + 4 \right] Y(s) = \left[6s^2 + 16s + 4 \right] X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{6s^2 + 16s + 4}{s^2 + 5s + 4}$$

1 mark for Laplace transforming the expression

2 marks for getting the transform right

2 marks for arranging into a transfer function

(b) **(7 marks)** A unit step input is applied to the system (i.e., $x(t) = u(t)$). What is the output of the system in the time domain?

$$\begin{aligned} Y(s) &= \frac{6s^2 + 16s + 4}{s^2 + 5s + 4} \times \frac{1}{s} \\ &= \frac{6s^2 + 16s + 4}{s(s+1)(s+4)} \\ &= \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4} \end{aligned}$$

Use coverup technique (or alternative) to find A , B and C .

$$\begin{aligned} A &= \left. \frac{6s^2 + 16s + 4}{(s+1)(s+4)} \right|_{s=0} = \frac{4}{4} = 1 \\ B &= \left. \frac{6s^2 + 16s + 4}{s(s+4)} \right|_{s \rightarrow -1} = \frac{-6}{-3} = 2 \\ C &= \left. \frac{6s^2 + 16s + 4}{s(s+1)} \right|_{s \rightarrow -4} = \frac{36}{12} = 3 \\ \Rightarrow Y(s) &= \frac{1}{s} + \frac{2}{s+1} + \frac{3}{s+4} \\ \Rightarrow y(t) &= \left[1 + 2e^{-t} + 3e^{-4t} \right] u(t) \end{aligned}$$

2 marks for finding $\mathcal{L}\{u(t)\} = \frac{1}{s}$, and multiplying with $G(s)$

4 marks for doing the partial fractions expansion correctly

1 mark for inverting to find $y(t)$

2. Transfer functions and partial fractions

(12 marks)

An input of $x(t) = 10 \sin(3t)$ is applied to a filter having transfer function

$$G(s) = \frac{1}{s+1}$$

(a) (3 marks) Show that the output $Y(s) = G(s)X(s)$ can be described by

$$Y(s) = \frac{30}{(s+1)(s^2+9)}$$

$$X(s) = \mathcal{L}\{10 \sin(3t)\}$$

$$= 10 \frac{3}{s^2+3^2}$$

$$= \frac{30}{s^2+9}$$

$$Y(s) = \frac{1}{s+1} \frac{30}{s^2+9}$$

$$= \frac{30}{(s+1)(s^2+9)}$$

2 marks for Laplace transforming $x(t)$ correctly.

1 mark for multiplying by $G(s)$

(b) (9 marks) Find $y(t)$

$$\begin{aligned}
 Y(s) &= \frac{30}{(s+1)(s^2+9)} \\
 &= \frac{A_1s + A_0}{s^2+9} + \frac{B}{s+1} \\
 B &= \left. \frac{30}{s^2+9} \right|_{s=-1} = \frac{30}{10} = 3
 \end{aligned}$$

$$\begin{aligned}
 \text{So, } Y(s) &= \frac{30}{(s+1)(s^2+9)} \\
 &= \frac{A_1s + A_0}{s^2+9} + \frac{3}{s+1} \\
 &= \frac{(A_1s + A_0)(s+1) + 3s^2 + 27}{(s+1)(s^2+9)} \\
 &= \frac{A_1s^2 + A_1s + A_0s + A_0 + 3s^2 + 27}{(s+1)(s^2+9)}
 \end{aligned}$$

$$\Rightarrow \begin{cases} s^2 & : A_1 + 3 = 0 \Rightarrow A_1 = -3 \\ s^1 & : A_1 + A_0 = 0 \Rightarrow A_0 = 3 \\ s^0 & : A_0 + 27 = 30 \end{cases}$$

$$\begin{aligned}
 Y(s) &= \frac{-3s + 3}{s^2+9} + \frac{3}{s+1} \\
 &= -\frac{3s-3}{s^2+9} + \frac{1}{s+1} \\
 &= -\frac{3s}{s^2+9} + \frac{3}{s^2+9} + \frac{1}{s+1}
 \end{aligned}$$

$$\Rightarrow y(t) = (-3 \cos(3t) + \sin(3t) + 3e^{-t}) u(t)$$

2 marks for correctly identifying the form of the partial fractions expansion.

6 marks for finding the correct coefficients (half marks if algebraic mistakes made along the way).

1 mark for correctly converting to the time domain (even if the answer is wrong because of mistakes made previously).

3. Partial fractions

(12 marks)

(a) (6 marks) Find the inverse Laplace transform of $G(s) = \frac{2}{(s+1)^2(s+2)}$.

$$\begin{aligned} G(s) &= \frac{2}{(s+1)^2(s+2)} \\ &= \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+2} \\ A &= \left. \frac{2}{s+2} \right|_{s=-1} = 2 \\ C &= \left. \frac{2}{(s+1)^2} \right|_{s=-2} = 2 \end{aligned}$$

There are multiple methods to find B .

$$\begin{aligned} B &= \lim_{s \rightarrow -1} \frac{d}{ds} \frac{2}{s+2} \\ &= \lim_{s \rightarrow -1} \frac{d}{ds} 2(s+2)^{-1} \\ &= \lim_{s \rightarrow -1} \frac{-2}{(s+2)^2} \\ &= -2 \end{aligned}$$

Or, substitute $s = 0$ (or some other value, $s \notin \{-1, -2\}$)

$$\begin{aligned} G(0) &= \frac{2}{2} = \frac{2}{1^2} + \frac{B}{1} + \frac{2}{2} \\ 1 &= 3 + B \\ \implies B &= -2 \end{aligned}$$

$$\begin{aligned} \text{Either way we have, } G(s) &= \frac{2}{(s+1)^2} - \frac{2}{s+1} + \frac{2}{s+2} \\ \implies g(t) &= \left[2te^{-t} - 2e^{-t} + 2e^{-2t} \right] u(t) \end{aligned}$$

2 marks for correctly identifying the partial fractions form

3 mark for finding the residuals

1 mark for converting to the time domain

- (b) **(6 marks)** A signal has an s-domain representation of $H(s) = \frac{s+6}{(s+2)^3}$.
What is $h(t)$?

$$H(s) = \frac{A}{(s+2)^3} + \frac{B}{(s+2)^2} + \frac{C}{s+2}$$
$$A = \lim_{s \rightarrow -2} s+6 = 4$$
$$B = \lim_{s \rightarrow -2} \frac{d}{ds} s+6 = \lim_{s \rightarrow -2} 1 = 1$$
$$C = \lim_{s \rightarrow -2} \frac{1}{2!} \frac{d}{ds} 1 = 0$$
$$\Rightarrow H(s) = \frac{4}{(s+2)^3} + \frac{1}{(s+2)^2}$$
$$\text{So, } h(t) = \left[\frac{4}{2} t^2 e^{-2t} + t e^{-2t} \right] u(t)$$
$$= (2t^2 + t) e^{-2t} u(t)$$

2 marks for correctly identifying the partial fractions form.

3 marks for finding the residuals.

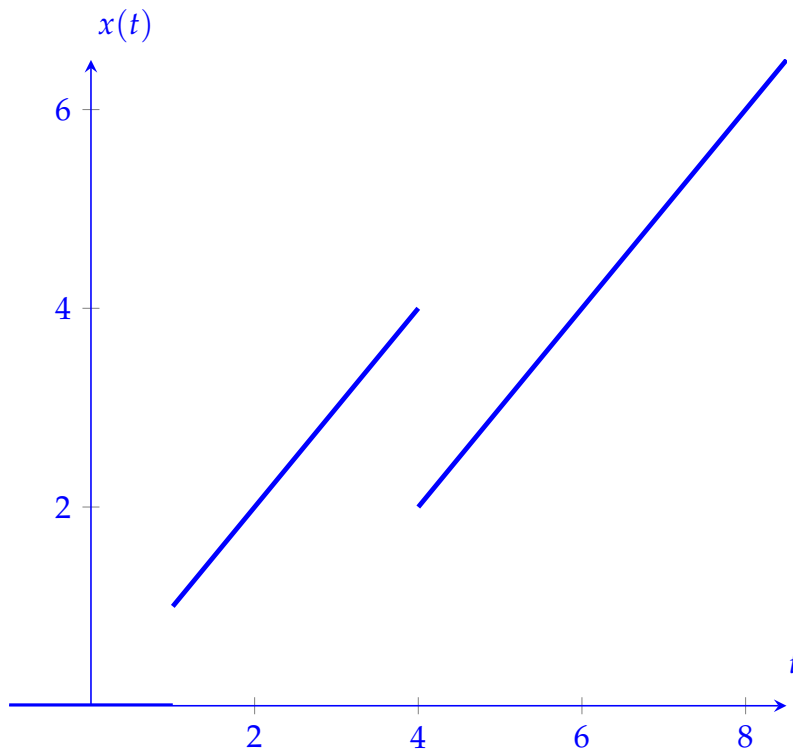
1 marks for converting to the time domain.

4. Unit steps

(14 marks)

(a) (3 marks)

Sketch the waveform of $x(t) = tu(t-1) - 2u(t-4)$



(b) (3 marks)

What is the Laplace transform of $x(t)$?

$$x(t) = (t-1)u(t-1) + u(t-1) - 2u(t-4)$$
$$\implies X(s) = \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} - \frac{2e^{-4s}}{s}$$

1 mark for rewriting the ramp part of the function in a form for which the time shift formula can be used (even if not written explicitly)

1 mark for correctly using the time shift formula to get the expected coefficients of s in the exponents.

1 mark for the correct denominators in $X(s)$.

(c) (8 marks)

If $x(t)$ is applied to a system having transfer function $G(s) = \frac{2}{s+2}$, what will the output be?

Warning: This question will take longer than reflected in the number of marks. Don't spend too long on this.

$$Y(s) = \frac{2}{s+2}X(s)$$

$$= \frac{2e^{-s}}{s^2(s+2)} + \frac{2e^{-s}}{s(s+2)} - \frac{4e^{-4s}}{s(s+2)}$$

Now, $\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$

$$A = \frac{1}{s+2} \Big|_{s=0} = \frac{1}{2}$$

$$B = \frac{1}{s} \Big|_{s=-2} = -\frac{1}{2}$$

$$Y(s) = \frac{2e^{-s}}{s^2(s+2)} + \frac{e^{-s}}{s} - \frac{e^{-s}}{s+2} - \frac{2e^{-4s}}{s} + \frac{2e^{-4s}}{s+2}$$

Similarly, $\frac{1}{s^2(s+2)} = \frac{E}{s^2} + \frac{F}{s} + \frac{G}{s+2}$

$$E = \frac{1}{s+2} \Big|_{s=0} = \frac{1}{2}$$

$$G = \frac{1}{s^2} \Big|_{s=-2} = \frac{1}{4}$$

$$\frac{1}{s^2(s+2)} = \frac{1}{2} \frac{1}{s^2} + \frac{F}{s} + \frac{1}{4} \frac{1}{s+2}$$

Substitute $s = 1$ (or something else)

$$\frac{1}{3} = \frac{1}{2} + F + \frac{1}{12}$$

$$\frac{4-6-1}{12} = F$$

$$F = -\frac{1}{4}$$

i.e., $\frac{1}{s^2(s+2)} = \frac{1}{2} \frac{1}{s^2} - \frac{1}{4} \frac{1}{s} + \frac{1}{4} \frac{1}{s+2}$

So finally, we get

$$Y(s) = \frac{e^{-s}}{s^2} - \frac{1}{2} \frac{e^{-s}}{s} + \frac{1}{2} \frac{e^{-s}}{s+2} + \frac{e^{-s}}{s} - \frac{e^{-s}}{s+2} - \frac{2e^{-4s}}{s} + \frac{2e^{-4s}}{s+2}$$

and, $y(t) = (t-1)u(t-1) - \frac{1}{2}u(t-1) + \frac{1}{2}e^{-2(t-1)}u(t-1)$
 $+ u(t-1) - e^{-2(t-1)}u(t-1) - 2u(t-4) + 2e^{-2(t-4)}$

1 mark for correctly identifying the partial fractions form
3 marks for finding the residuals
1 marks for converting to the time domain

* * * * *

Surname:

First Name:

Student ID:



ASSIGNMENT 1 – 2020

TRIMESTER 1

ECEN 203
ANALOGUE CIRCUITS
AND SYSTEMS

Time Allowed: FIFTY MINUTES

CLOSED BOOK

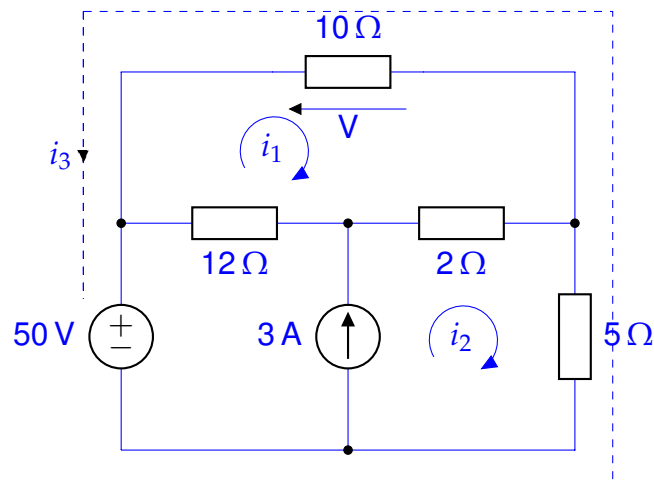
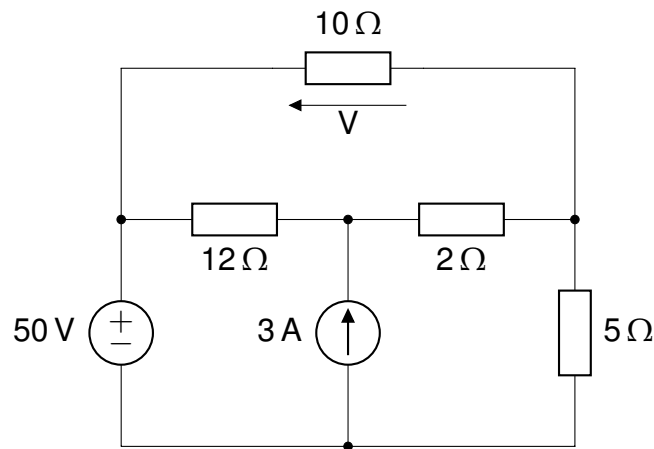
Permitted materials: You may use a scientific calculator, but not a smartphone or other similar device.

Instructions: Attempt ALL THREE Questions
Write your answers on the question paper.
Full marks will be awarded for correct answers.
Partial marks may be awarded to incorrect answers if working is shown.
The exam will be marked out of a total of 50 marks.

1. Mesh current analysis

(20 marks)

Find V by mesh current analysis. Note: you will need to come up with a strategy to deal with the current source.



Remember that the key is to make sure that only one current flows through the current source, hence the need for the loop around the outside, or perhaps around the bottom half.

(4 marks)

$$\begin{aligned}
 -10i_1 - 2i_1 - 12i_1 + 2i_2 + 10i_3 &= 0 \\
 i_2 &= 3 \\
 10i_1 + 5i_2 - 5i_3 - 10i_3 - 50 &= 0
 \end{aligned}$$

(6 marks)

$$\begin{aligned}
 -24i_1 + 10i_3 &= -6 \\
 10i_1 - 15i_3 &= 35
 \end{aligned}$$

$$12i_1 - 5i_3 = 3$$

$$2i_1 - 3i_3 = 7$$

$$\begin{bmatrix} 12 & -5 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

(2 marks)

$$\begin{vmatrix} 12 & -5 \\ 2 & -3 \end{vmatrix} = (12)(-3) - (2)(-5) = -36 + 10 = -26$$
$$\begin{vmatrix} 3 & -5 \\ 7 & -3 \end{vmatrix} = (3)(-3) - (7)(-5) = -9 + 35 = 26$$
$$\begin{vmatrix} 12 & 3 \\ 2 & 7 \end{vmatrix} = (12)(7) - (2)(3) = 84 - 6 = 78$$

(3 marks)

So

$$i_1 = \frac{26}{-26} = 1 \text{ A}$$

$$i_3 = \frac{78}{-26} = -3 \text{ A}$$

(2 marks)

So the current through the 10Ω resistor is $i_1 - i_3 = -1 - -3 = 2 \text{ A}$ to the right.

(2 marks)

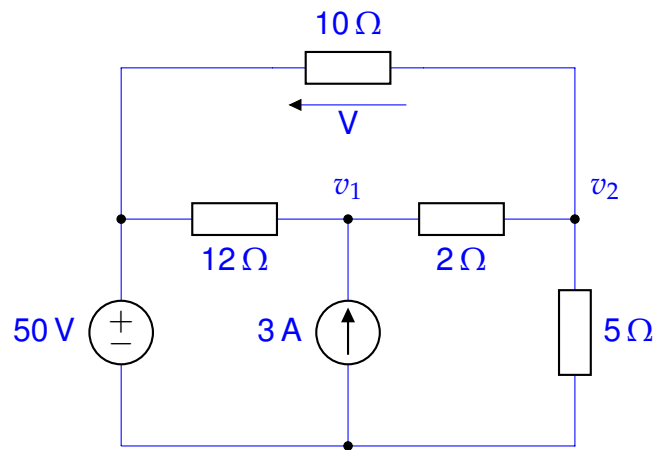
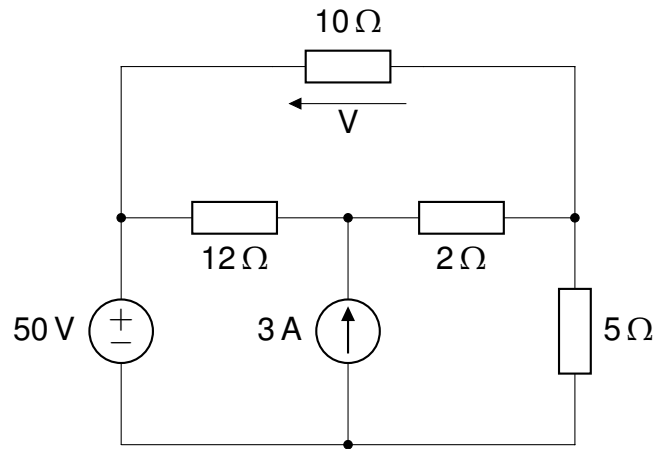
$$\text{So } V = (10)(2) = +20 \text{ V}$$

(1 mark)

2. Node voltage analysis

(15 marks)

Find V by node voltage analysis. (Note: this is the same circuit as the previous question).



(2 marks)

At node 1:

$$\frac{v_1 - 50}{12} - 3 + \frac{v_1 - v_2}{2} = 0$$

(3 marks)

At node 2:

$$\frac{v_2 - v_1}{2} + \frac{v_2}{5} + \frac{v_2 - 50}{10} = 0$$

(3 marks)

$$\begin{bmatrix} \frac{7}{12} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{8}{10} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{86}{12} \\ 5 \end{bmatrix}$$

(2 marks)

$$\begin{bmatrix} 7 & -6 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 86 \\ 50 \end{bmatrix}$$

Note that we don't need to solve this for v_1 , just v_2 :

$$\begin{aligned} v_2 &= \frac{\begin{vmatrix} 7 & 86 \\ -5 & 50 \end{vmatrix}}{\begin{vmatrix} 7 & -6 \\ -5 & 8 \end{vmatrix}} = \frac{350 + 430}{56 - 30} \\ &= \frac{780}{26} \\ &= 30 \end{aligned}$$

(3 marks)

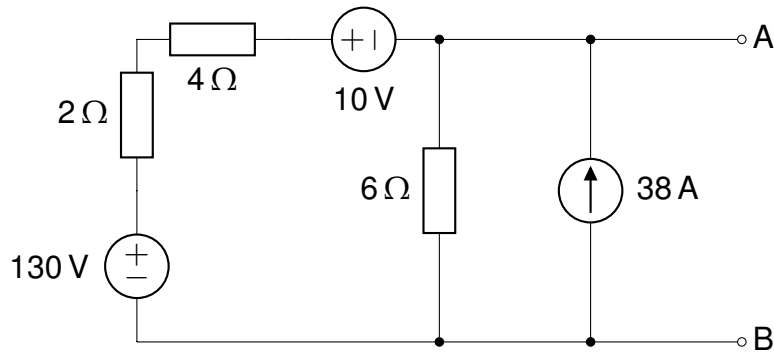
$$\text{So } v = 50 - 30 = 20 \text{ V}$$

(2 marks)

3. Thevenin/Norton equivalents

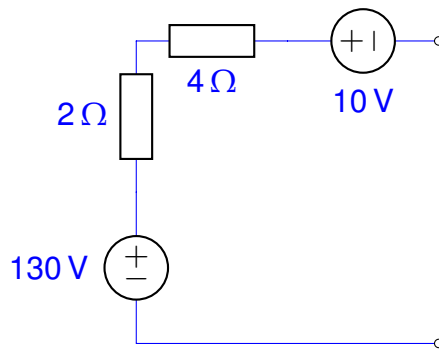
(15 marks)

Find the Thevenin equivalent at terminals A&B:

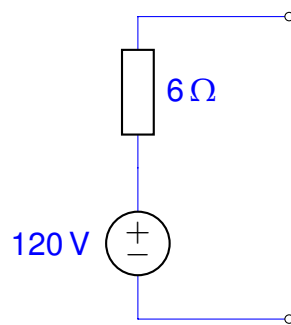


There are many ways to do this.

One of the easiest is to replace:

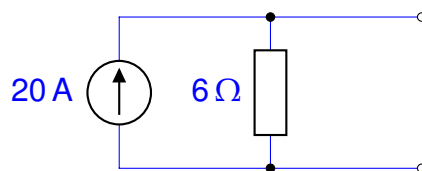


by



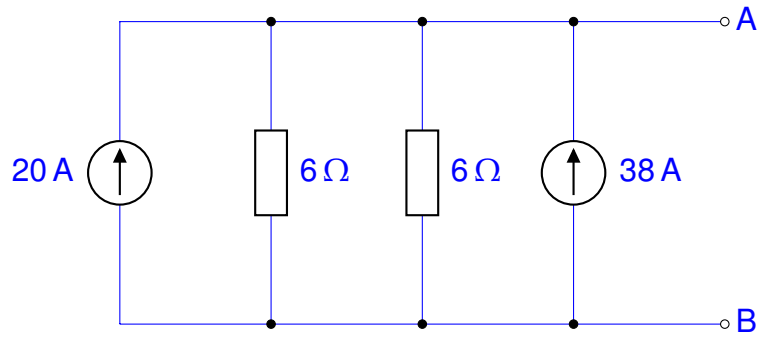
(5 marks)

and then by its Norton equivalent



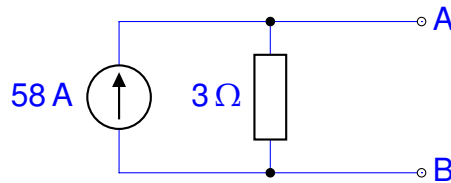
(3 marks)

And then we have



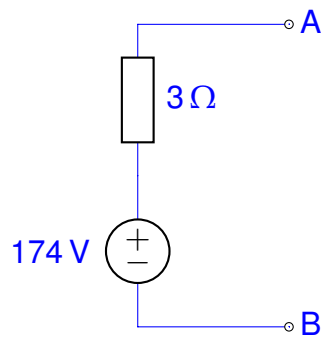
(2 marks)

and hence



(3 marks)

and so the Thevenin equivalent is



(2 marks)

It is actually very easy to get the Thevenin resistance, by zeroing the sources to get $6 \parallel 6 = 3 \Omega$, but finding the open-circuit voltage or the short-circuit current is possibly a bit harder than the transformations above.

Surname:

First Name:

Student ID:



VICTORIA UNIVERSITY OF
WELLINGTON
TE HERENGA WAKA

CLASS TEST 2 – 2020

TRIMESTER 1

ECEN 203
ANALOGUE CIRCUITS
AND SYSTEMS

Time Allowed: THIRTY TWO HOURS

OPEN BOOK

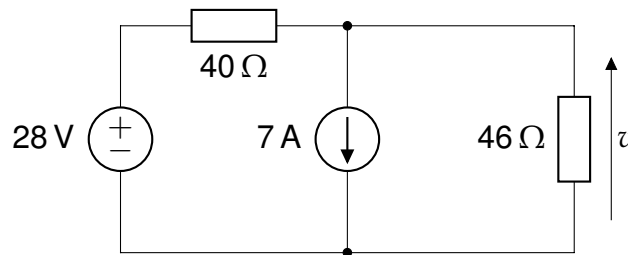
Permitted materials: Any.

Instructions: Attempt ALL Questions
Full marks will be awarded for correct answers.
Partial marks may be awarded to incorrect answers if working is shown.
Along with your worked answers, please submit a text file with the name answers.txt, which has one numerical answer per line.
The test will be marked out of a total of 50 marks.

Student ID:

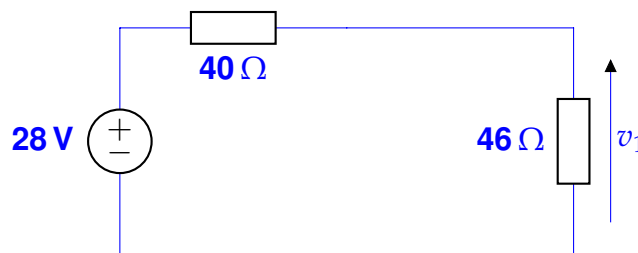
1. DC Circuits

(5 marks)



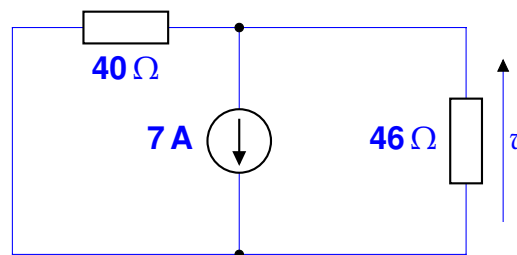
Use the principle of superposition to find the voltage v (in volts).

We first include just the voltage source



For which we obtain $v_1 = \frac{(28)(46)}{40+46} = 14.9767$ Volts

Then we include just the current source

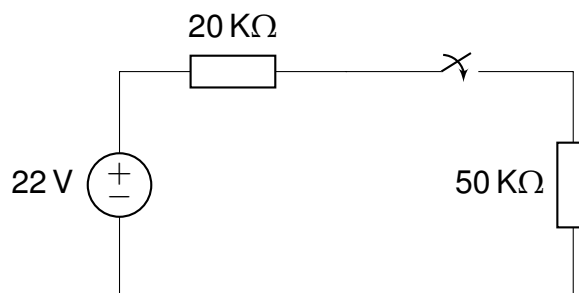


For this we have $v_2 = -7 \frac{40 \times 46}{40+46} = -149.7674$

So the required voltage is $v_1 + v_2 = 14.9767 + -149.7674 = -134.7907$

2. DC circuits

(2 marks)

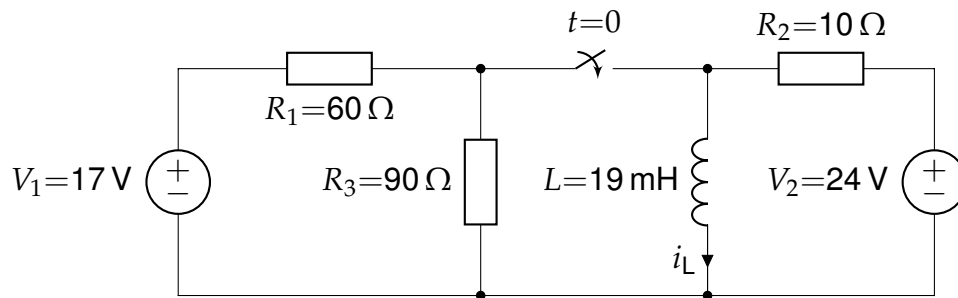


What is the voltage (in volts) between the terminals of the switch before it is closed?

There is no current flowing in the resistors, so zero voltage drop across them, so the voltage across the switch is simply the source voltage. i.e., $V_{\text{switch}} = 22$.

3. Transient analysis

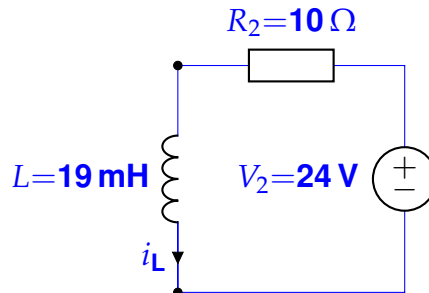
(20 marks)



The switch has been open for a long time. Then at time $t = 0$ the switch is closed.

- (a) (2 marks) What is the value (in amperes) of the inductor current i_L at time $t = 0^+$

While the switch is open, the only relevant part of the circuit is the right part:



At steady state, the inductor behaves as a short circuit, and so the current is $\frac{V_2}{R_2} = \frac{24}{10} = 2.4000 \text{ A}$

- (b) (10 marks) What will be the final value (in amperes) of the inductor current? (Hint: Start by using Norton's and Thevenin's theorems).

Once the switch is closed, we can analyse the behaviour by finding the Thevenin equivalent of all parts of the circuit except for the inductor.

The left part has $V_T = \frac{V_1 R_3}{R_1 + R_3} = \frac{17 \times 90}{60 + 90} = 10.2000 \text{ V}$

and $R_T = R_1 \parallel R_3 = \frac{R_1 R_3}{R_1 + R_3} = \frac{60 \times 90}{60 + 90} = 36.0000 \Omega$

The Norton equivalent of this is a current source of $I_1 = V_T / R_T = 10.2000 / 36.0000 = 0.2833 \text{ A}$ in parallel with $R_T = 36.0000 \Omega$

The Norton equivalent of the right part of the circuit is $I_2 = \frac{V_2}{R_2} = \frac{24}{10} = 2.4000 \text{ A}$ in parallel with $R_2 = 10 \Omega$

So the combined Norton equivalent is $I_3 = I_1 + I_2 = 0.2833 + 2.4000 = 2.6833 \text{ A}$ in parallel with $R_{N3} = R_2 \parallel R_T = \frac{R_2 R_T}{R_2 + R_T} = 7.8261 \Omega$, and the Thevenin equivalent is the same resistance in series with $V_3 = 2.6833 \times 7.8261 = 21.0000 \text{ V}$

Once the inductor is again in its steady state (behaving as a short circuit), the current it carries will be $I_3 = 2.6833 \text{ A}$

- (c) **(3 marks)** What is the time constant (in milliseconds) of the change in the inductor current?

The time constant is $\tau = L/R_{N3} = 2.4278 \text{ ms}$

- (d) **(5 marks)** What will be the inductor current (in amperes) at time $t = 1 \text{ ms}$?

The formula for the current will be

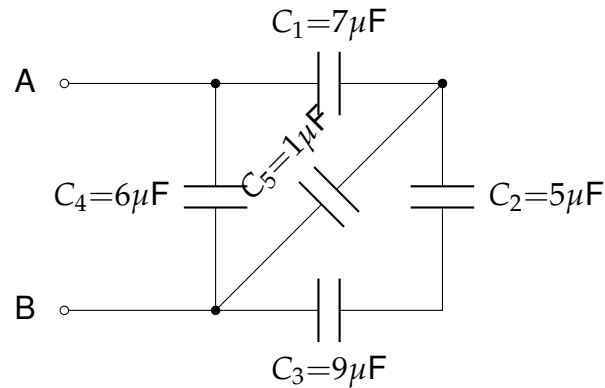
$$i_L(t) = I_{\text{final}} + (I_{\text{initial}} - I_{\text{final}})e^{-\frac{t}{\tau}}$$

If we plug in the values we get

$$\begin{aligned}i_L(1 \text{ ms}) &= 2.6833 + (2.4000 - 2.6833)e^{-\frac{1}{2.4278}} \\ &= 2.4957\end{aligned}$$

4. Capacitor Circuits

(5 marks)



What is the capacitance between terminals A and B (in microfarads)?

It's probably easier to first express this if we first think of these as impedances:

$$Z = ((Z_2 + Z_3) \parallel Z_5 + Z_1) \parallel Z_4$$

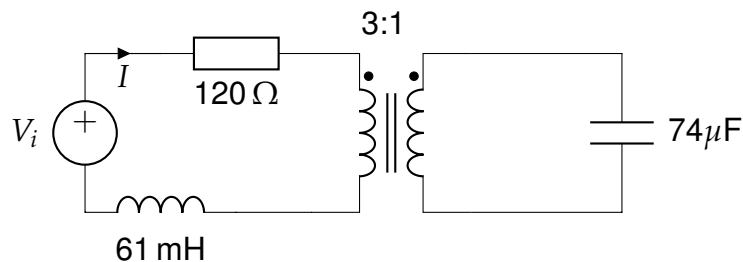
But because capacitors add in parallel and the reciprocals add when they are in series, we have

$$C = (((C_2 \parallel C_3) + C_5) \parallel C_1) + C_4$$

Plugging in the numbers we get $C = 8.6306 \mu\text{F}$

5. AC Analysis

(7 marks)



V_i is a sinusoidal AC voltage source having frequency 100 Hz, amplitude 5 V and phase 0.

(a) (5 marks) What is the amplitude (in milliamperes) of the current I ?

First note that the capacitor is symmetrical, so the position of the dot makes no difference for this question.

The angular frequency is $\omega = 2 * \pi * f = 628.3185 \text{ rad s}^{-1}$

The impedance of the capacitor is $Z_C = \frac{1}{j\omega C} = -21.5074j \Omega$

But this is transformed by the square the turns ratio to $Z_{C2} = 3^2 \times (-21.5074j) = -193.5668j$

The impedance of the inductor is $Z_L = j\omega L = 38.3274j \Omega$

So the total impedance of the resistor, inductor and (transformed) capacitor is $Z = Z_L + Z_{C2} + R = 120.0000 + -155.2394j \Omega$

The current is $V_i/Z = 5/(120.0000 + -155.2394j) = 0.0156 + 0.0202j \text{ A}$

This has magnitude 25.4826 mA

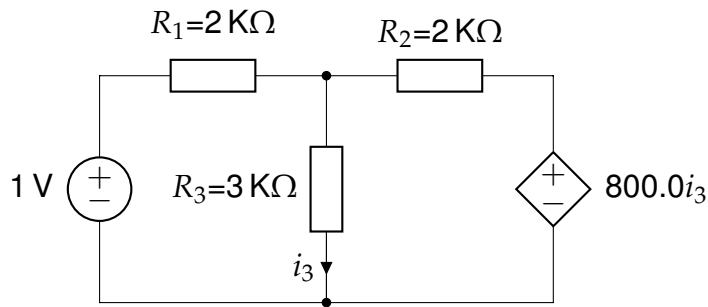
(b) (2 marks)

What is the phase (in degrees) of the current I in part (a)?

And phase 52.2960°

6. Controlled sources

(6 marks)



What is the voltage (in volts) of the controlled source?

Probably easiest to use the node voltage approach. Putting node v between R_1 and R_2 , we apply KCL to get:

$$\frac{v - V_1}{R_1} + \frac{v}{R_3} + \frac{v - ki_3}{R_2} = 0$$

we also can see that $i_3 = \frac{v}{R_3}$. We combine these to get

$$\frac{v - V_1}{R_1} + \frac{v}{R_3} + \frac{v - kv/R_3}{R_2} = 0$$

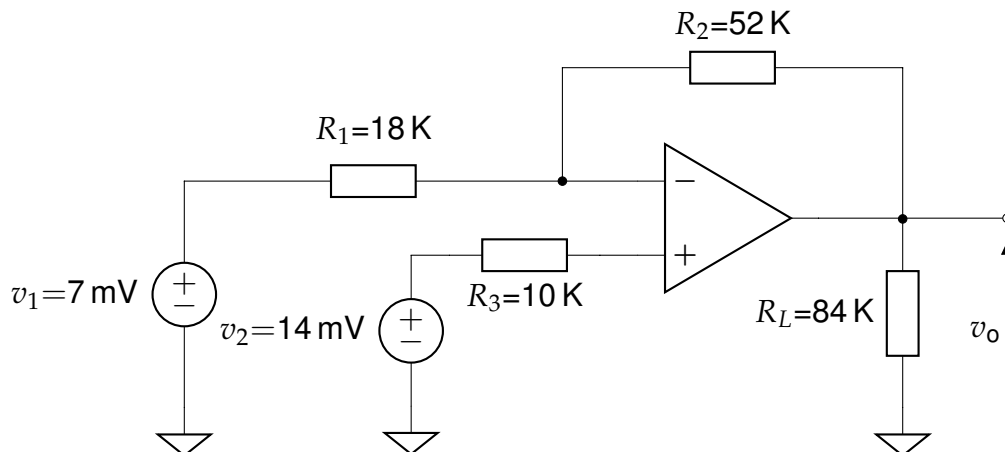
$$v \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} - \frac{k}{R_2 R_3} \right) = \frac{V}{R_1}$$

Plugging in the numbers we get $v = 0.4167 \text{ V}$

We need to divide this by R_3 to get i_3 and then multiply to k to get the voltage across the controlled source: 0.1111 V

7. Op-amps

(5 marks)



Assuming the op-amp is ideal, what is the value of v_o (in millivolts)?

First note that the value of R_3 is irrelevant, because (almost) no current flows through it. So $v_- = v_2$. Also, we have negative feedback, and hence a virtual short, so $v_+ = v_- = v_2$

We then have $\frac{v_o - v_2}{R_2} = \frac{v_2 - v_1}{R_1}$ which we can rearrange to give $v_o = \left(1 + \frac{R_2}{R_1}\right) v_2 - \frac{R_2}{R_1} v_1$

Substituting in the numbers, we get $v_o = 34.2222 \text{ mV}$

Surname:

First Name:

Student ID:



VICTORIA UNIVERSITY OF
WELLINGTON
TE HERENGA WAKA

ASSIGNMENT 1 – 2021

TRIMESTER 1

EEEN 203
ANALOGUE CIRCUITS
AND SYSTEMS

Time Allowed: FIFTY MINUTES

CLOSED BOOK

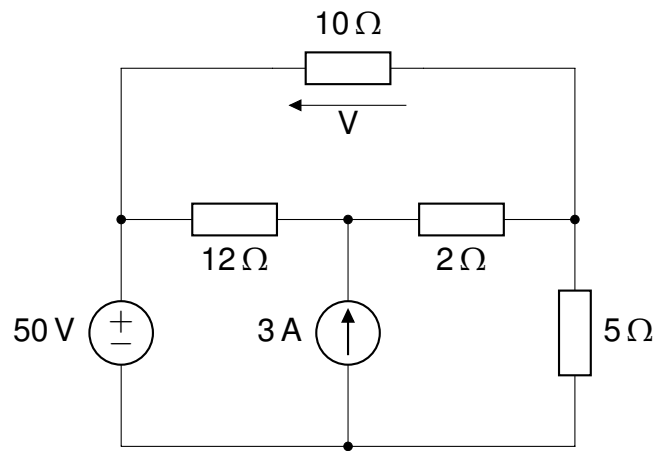
Permitted materials: You may use a scientific calculator, but not a smartphone or other similar device.

Instructions: Attempt Questions 1–3.
Attempt Question 4 ONLY if you have time and are confident about your answers to Questions 1–3.
Write your answers on the question paper.
Full marks will be awarded for correct answers.
Partial marks may be awarded to incorrect answers if working is shown.

1. Mesh current analysis

(20 marks)

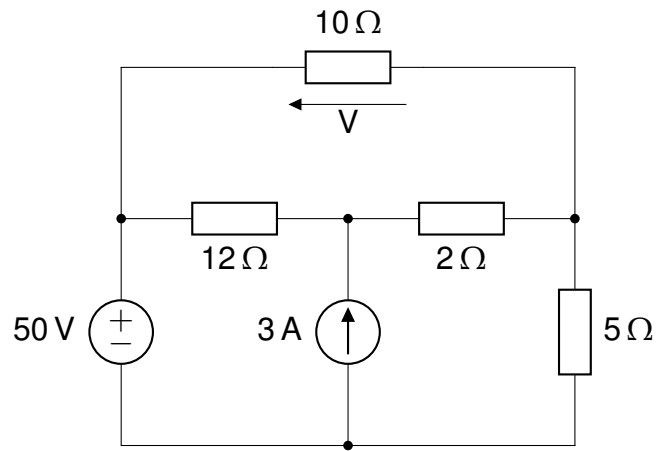
Find V by mesh current analysis. Note: you will need to come up with a strategy to deal with the current source.



2. Node voltage analysis

(15 marks)

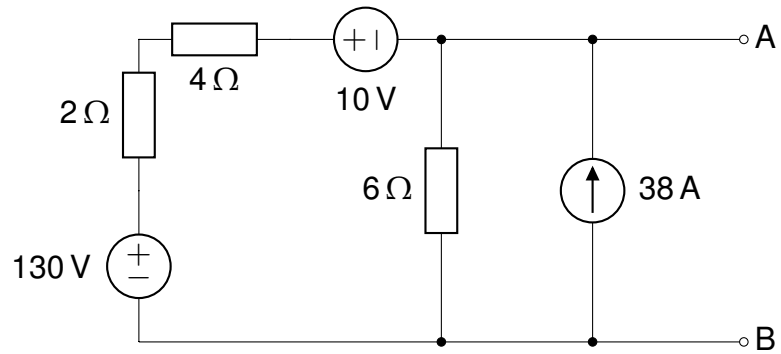
Find V by node voltage analysis. (Note: this is the same circuit as the previous question).



3. Thevenin/Norton equivalents

(15 marks)

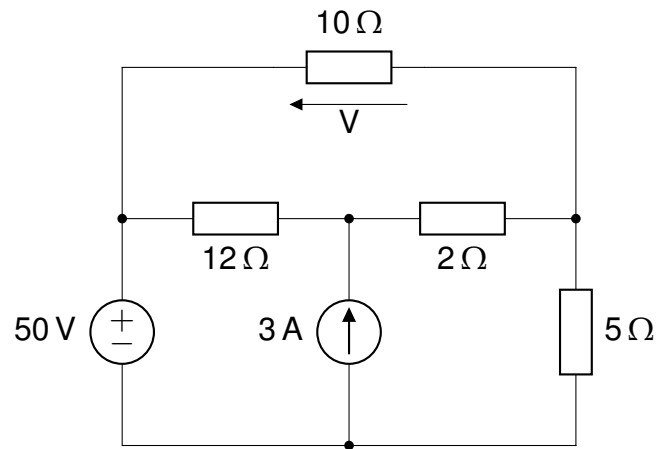
Find the Thevenin equivalent at terminals A&B:



4. Superposition

(15 marks)

Find V by using the principle of superposition. (Note: this is the same circuit as questions 1 and 2).



* * * * *

Surname:

First Name:

Student ID:



VICTORIA UNIVERSITY OF
WELLINGTON
TE HERENGA WAKA

CLASS TEST 2 – 2021

TRIMESTER 1

ECEN 203
ANALOGUE CIRCUITS
AND SYSTEMS

Time Allowed: FIFTY MINUTES

CLOSED BOOK

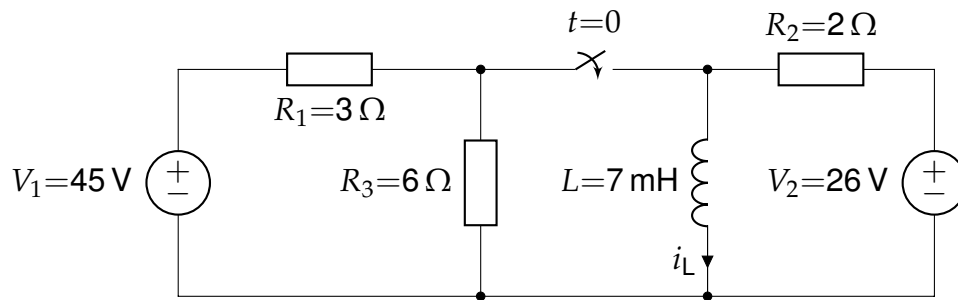
Permitted materials: Calculators.

Instructions: Attempt ALL Questions
Full marks will be awarded for correct answers.
Partial marks may be awarded to incorrect answers if working is shown.

The test will be marked out of a total of 50 marks.

Student ID:

1. Transient analysis

(24 marks)

The switch has been open for a long time. Then at time $t = 0$ the switch is closed.

(a) **(4 marks)** What is the value (in amperes) of the inductor current i_L at time $t = 0^+$ (Hint: Only part of the circuit is relevant for finding this value).

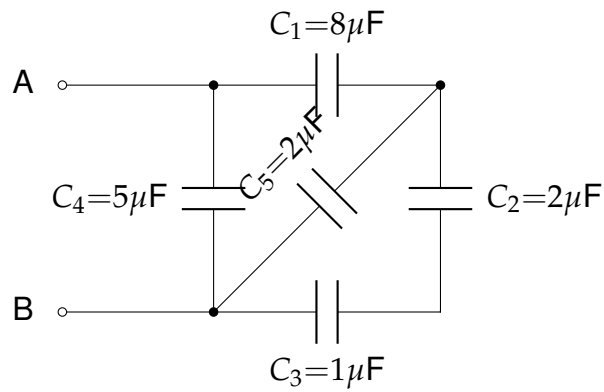
(b) **(10 marks)** What will be the final value (in amperes) of the inductor current? (Hint: Start by using both Norton's and Thevenin's theorems).

(c) **(4 marks)** What is the time constant (in milliseconds) of the change in the inductor current?

(d) **(6 marks)** What will be the inductor current (in amperes) at time $t = 1 \text{ ms}$?

2. Capacitor Circuits

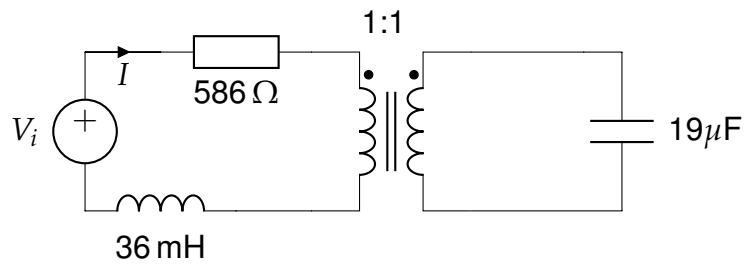
(6 marks)



What is the capacitance between terminals A and B (in microfarads)?

3. AC Analysis

(10 marks)



V_i is a sinusoidal AC voltage source having frequency 110 Hz, amplitude 90 V and phase 0.

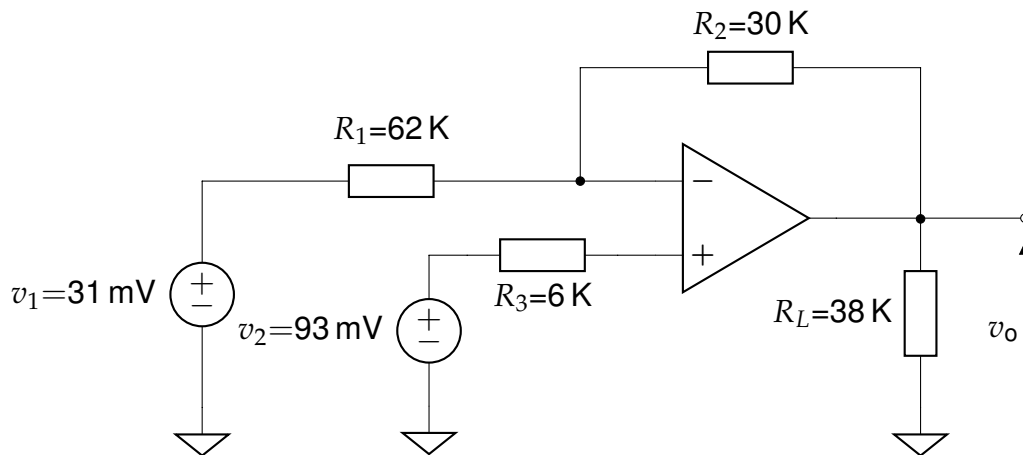
(a) (5 marks) What is the amplitude (in milliamperes) of the current I ?

(b) (5 marks)

What is the phase (in degrees) of the current I in part (a)?

4. Op-amps

(10 marks)



Assuming the op-amp is ideal, what is the value of v_o (in millivolts)?

2022 EEEN 203 Circuit Analysis

Test 4

Total marks: 85 (Worth 16%)

Below are the problems for this assignment. Do your calculation as needed and then put your final answers as well any discussion or plots in the spaces required. Submit this document with a filename:

EEEN203_Test4_2022_”your surname”-”your initial” on the Wiki submission system no later than Saturday 13 June by 8.00 pm.

Name:

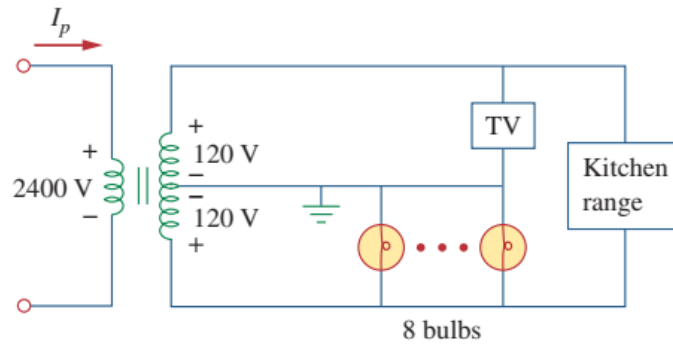
Student Number:.....

Q1. An ideal transformer is rated at 2400/220 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Answer

(10)

Q2. A distribution transformer is used to supply a household as in Figure. The load consists of eight 100-W bulbs, a 350-W TV, and a 15-kW kitchen range. If the secondary side of the transformer has 72 turns, calculate: (a) the number of turns of the primary winding, and (b) the current I_p in the primary winding.



Answer

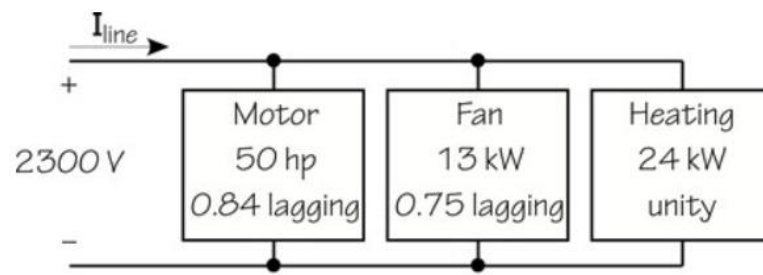
(10)

Q3. The voltage $v = 12 \cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Answer

(6)

Q4. Find Real, Reactive and Apparent power for following load configuration.



Answer

(15)

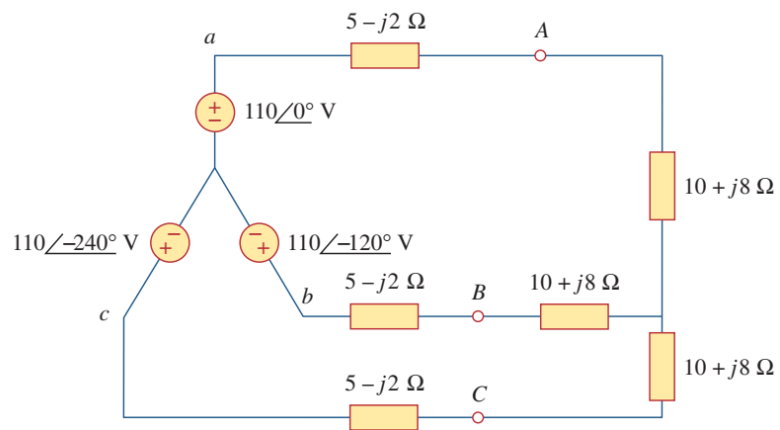
Blank area for the answer.

Q5. When connected to a 240-V (rms), 50-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Answer

(20)

Q6. Determine the total average, reactive, and complex powers at the source and at the load for the following figure.



Answer

(24)

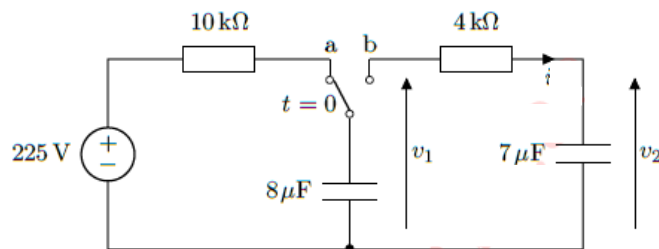
EEEN203: Circuit Analysis

Test 2

8:00 p.m., Monday 24 April 2023

RC Circuits

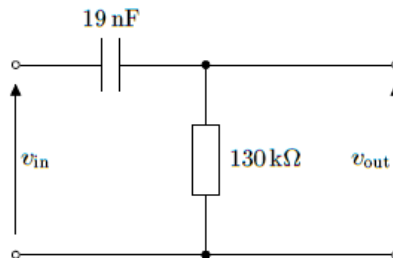
1. The switch in the circuit has been in position a for a long time, and $v_2(0) = 0$. At time $t = 0$ the switch is moved to position b.



- (a) Find the voltage $v_1(0^+)$ (in volts) [1 mark]
- (b) Find the voltage $v_2(0^+)$ (in volts) [1 mark]
- (c) Find the current $i(0^+)$ (in milliamperes) [1 mark]
- (d) Find the energy stored in the capacitors at $t = 0$ (in millijoules) [1 mark]
- (e) Find the voltage $v_1(\infty)$ (in volts) (Hint: The answer is not zero; why not?) [3 marks]
- (f) Find the total energy dissipated in the $4\text{ k}\Omega$ resistor (in millijoules). [1 mark]

Filters

2.



- (a) Find the cutoff frequency f_c (in Hz) for the RC filter [1 mark]
- (b) Calculate $|H(j\omega)|$ at $\omega = 0.75\omega_c$ where ω_c is the cut-off frequency of the filter in rad s^{-1} [1 mark]

(c) Calculate $\angle H(j\omega)$ at $\omega = 1.12\omega_c$ (in degrees)

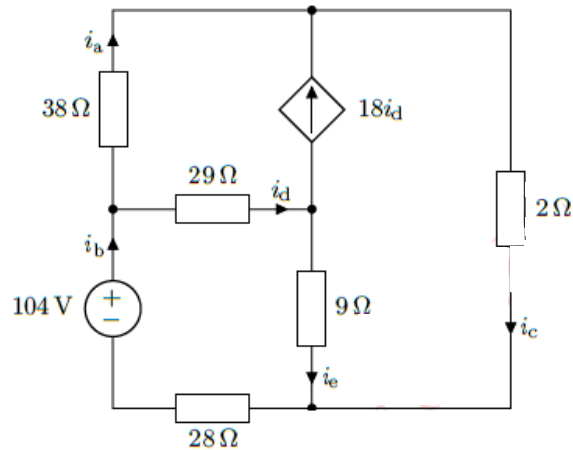
[1 mark]

(d) If $v_{in}(t) = 24 \cos(6376t)$, what is $v_{out}(4.025)$ (in volts)?

[1 mark]

Controlled Sources

3. Use the mesh-current method to analyse the circuit below:



(a) Find the branch current i_a (in amperes)

[1 mark]

(b) Find the branch current i_b (in amperes)

[1 mark]

(c) Find the branch current i_c (in amperes)

[1 mark]

(d) Find the branch current i_d (in amperes)

[1 mark]

(e) Find the branch current i_e (in amperes)

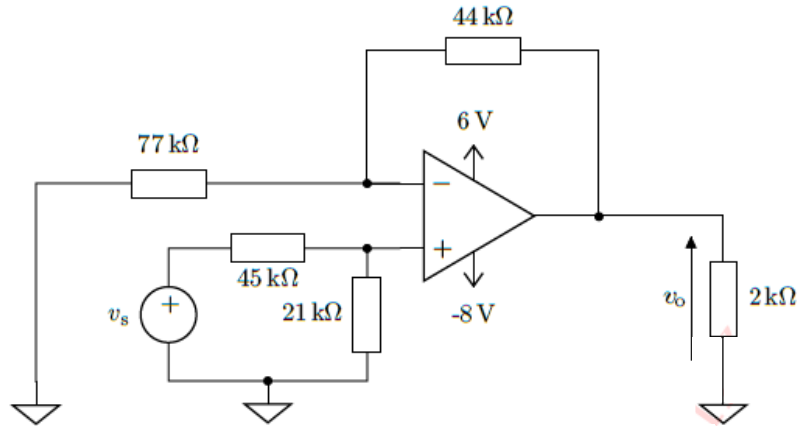
[1 mark]

(f) Find the total power developed in the circuit (in watts).

[2 marks]

Op-amp circuits

4. The op-amp in the circuit below is ideal.



- (a) Find the gain of the circuit from v_s to v_o . [1 mark]
- (b) Find the minimum value of v_s (in volts) for which the op-amp remains in its linear region of operation (in volts). [1 mark]
- (c) Find the maximum value of v_s (in volts) for which the op-amp remains in its linear region of operation (in volts). [1 mark]

Total marks: 90 (Worth 17%)

Below are seven problems for this test. Do your calculations as needed and then put your answers (both working and final) as well any discussion or plots in a new document.

Submit your answers with a filename:

EEEN203_Test4_2023_” your surname”-“your initial” on the Wiki submission system no later than Monday 12 June by 8.00 pm

Note: You are allowed to work in a team and share knowledge but let me know who you worked with so that I don't penalise if the two solutions are similar.

Name:

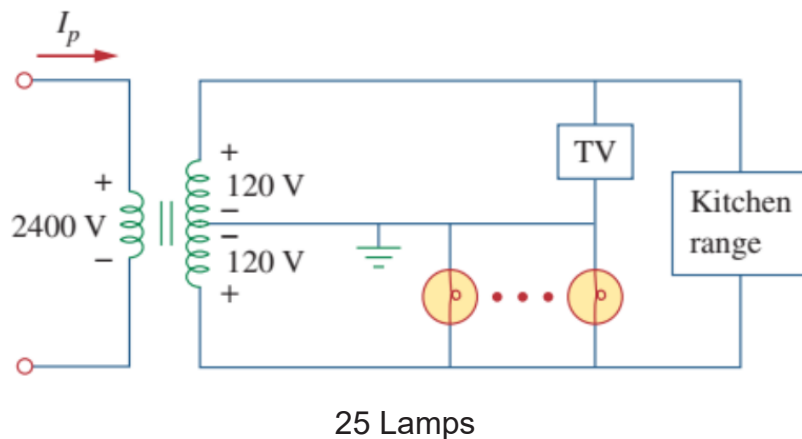
Student Number:.....

Q1. An ideal transformer is rated at 2400/220 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

[10 Marks]

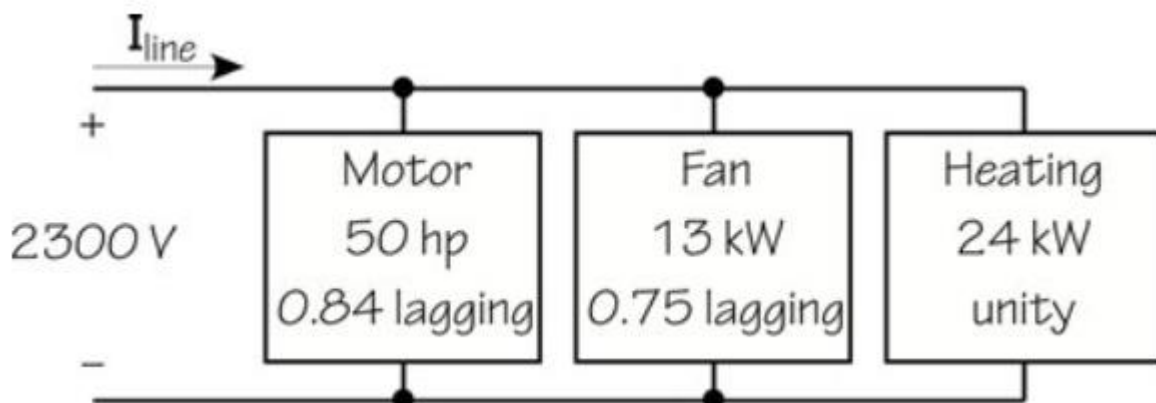
Q2. A distribution transformer is used to supply a household as in Figure. The load consists of twenty-five 100-W bulbs, a 350-W TV, and a 3.5-kW kitchen range. If the secondary side of the transformer has 72 turns, calculate: (a) the number of turns of the primary winding, and (b) the current I_p in the primary winding.

[10 Marks]



Q3. Find Real, Reactive and Apparent Power for the following load configuration.

[15 Marks]



Q4. When connected to a 240-V (rms), 50-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.6. Find the value of capacitance necessary to raise the power factor to 0.95. **[15 Marks]**

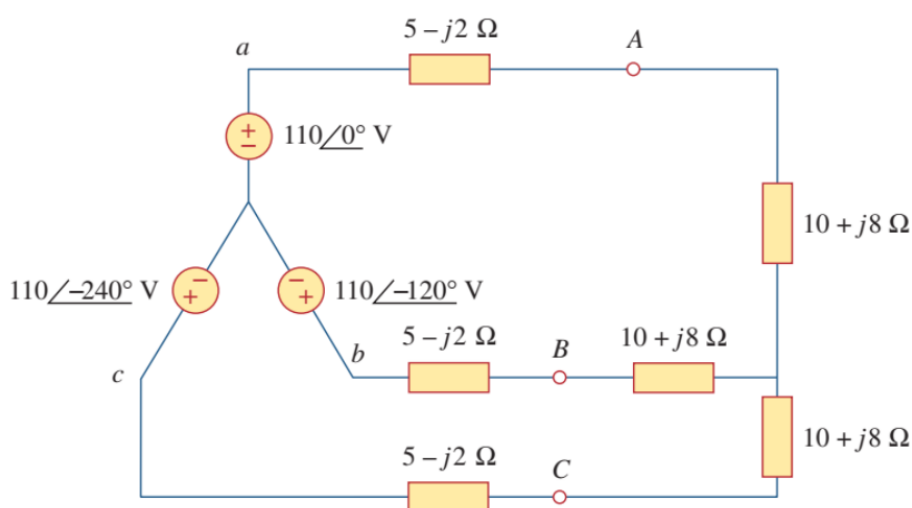
Q5. A load Z draws 15 kVA at a power factor of 0.7 lagging from a 240-V rms sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance. **[10 Marks]**

Q6. A 1200-kW load supplied at 13 kV (rms) operates 410 hours a month at 76 percent power factor. Calculate the average cost per month based on this simplified tariff:

- Energy charge: 8 cents per kWh
- Power-factor penalty: 0.2 percent of energy charge for every 0.01 that pf falls below 0.85.
- Power-factor credit: 0.24 percent of energy charge for every 0.01 that pf exceeds 0.85.

[10 Marks]

Q7. Determine the total average, reactive, and complex powers at the source and at the load for the following figure. **[20 Marks]**





TEST 1
TRIMESTER 1 2024

EEEN203 CIRCUIT ANALYSIS

CRN 33055

Time allowed : One Hour

Instructions : Write your answers in the answer booklet provided.
To get a full score, you must provide detail steps.
Not all questions have equal marks.
Dictionary and non-programmable calculator are permitted.
There are FIVE questions. Answer ALL questions.

Question 1:	Nodal Analysis and Thevenin Equivalent.	20 Marks
Question 2:	Inductor Circuit.	8 Marks
Question 3:	AC Circuit Parameters.	8 Marks
Question 4:	Filter Circuit.	6 Marks
Question 5:	Op Amp Amplifiers.	8 Marks

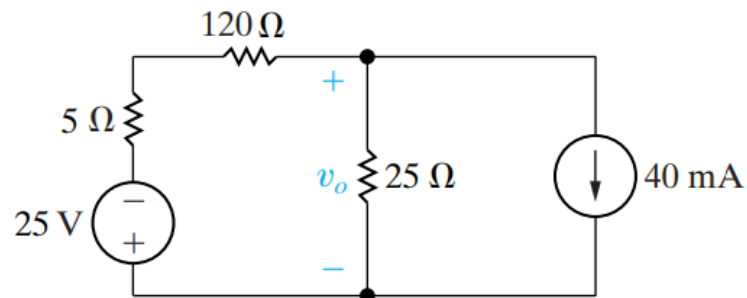
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Section A. DC Circuit Analysis

[20 Marks]

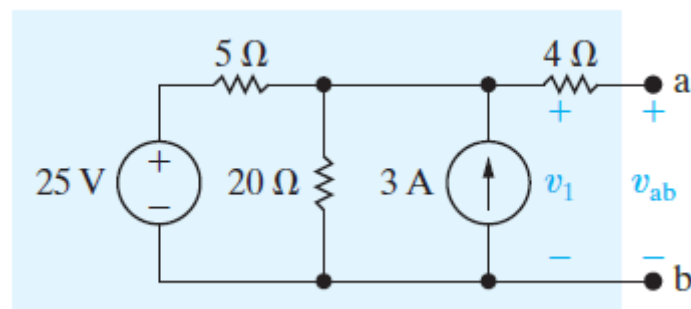
1. Use the suggested circuit analysis methods to calculate the stated unknown parameters of the circuit.

a. Applying node-voltage method, find v_o in the circuit given in the figure below.
[6 marks]



b. Find the Thevenin equivalent of the circuit given in the figure below.

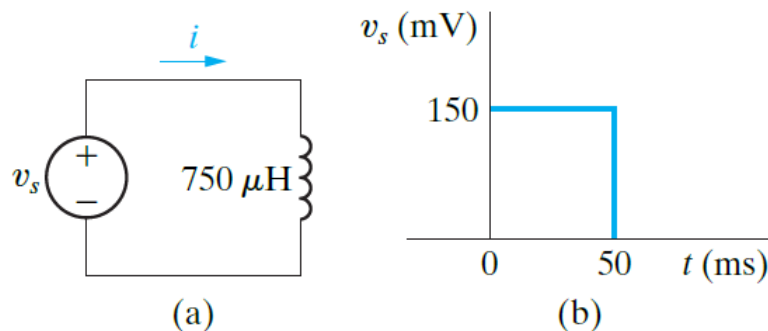
[14 Marks]



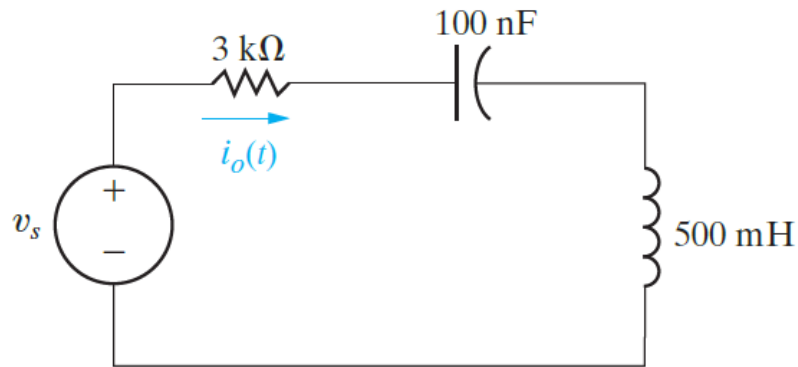
Section B. AC Circuit Analysis

[22 Marks]

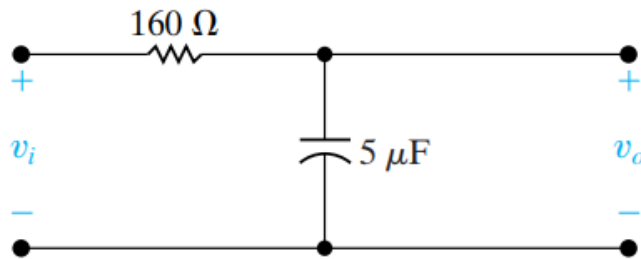
2. The voltage at the terminals of the $750 \mu\text{H}$ inductor circuit in figure (a) is shown in the figure (b). The inductor current $i(t)$ is known to be zero for $t < 0$. Derive the expressions for $i(t)$ when $t > 0$.
[8 marks]



3. Find the steady-state expression for $i_o(t)$ in the circuit in the figure below if $v_s = 80 \cos 2000t$ V. [8 marks]



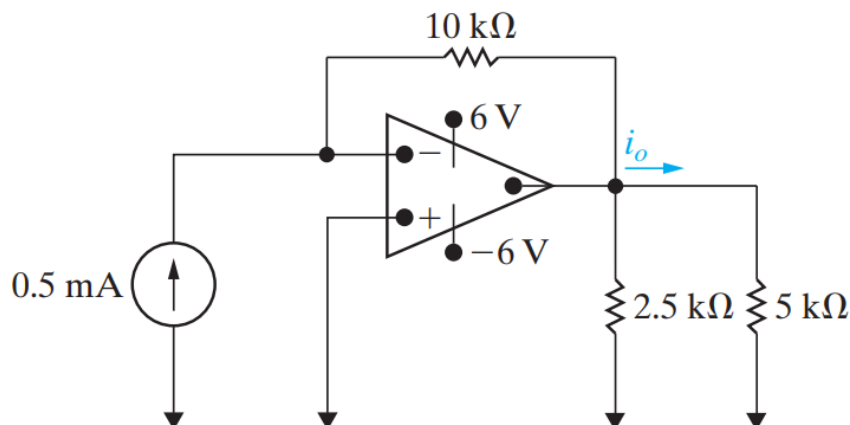
4. Using the filter circuit shown in the figure below, find the cut-off frequency (in Hertz) of the filter. Describe briefly the characteristic of the filter. [6 Marks]



Section C. Operational Amplifiers

[8 Marks]

5. Considering ideal op amp used in the circuits below, the voltage at the inverting pin is equal to the voltage at the noninverting pin ($v_n = v_p$). Find the current through 5 kΩ resistor i_o . If a practical op amp is used instead in the circuit, describe briefly its gain and input/output impedance characteristics and how these influence the circuit. [8 marks]



USEFUL FORMULAE

A. Basic Electricals

For Capacitor:

$$E = \frac{1}{2} QV \quad Q = CV \quad C = \frac{\epsilon_0 \epsilon_r A}{d} \quad C_T = C_1 + C_2 + \dots \quad \tau = RC$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

For Inductor:

$$\phi = BA \quad \mathcal{E} = -L \left(\frac{\Delta I}{dt} \right) \quad \mathcal{E} = - \frac{\Delta \phi}{dt} \quad E = \frac{1}{2} LI^2 \quad \tau = \frac{L}{R}$$

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

Electrical Parameters:

$$I = I_{max} \sin \omega t \quad V = V_{max} \sin \omega t \quad I_{max} = \sqrt{2} I_{rms} \quad V_{max} = \sqrt{2} V_{rms} \quad X_c = \frac{1}{\omega C}$$

$$X_L = \omega L \quad V = IZ \quad \omega = 2\pi f \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

B. Circuit Theorems

Kirchhoff Voltage Law (KVL)

$$\sum_{i=1}^n V_i = V_1 + V_2 + V_3 + \dots = 0$$

Kirchhoff Current Law (KCL)

$$\sum_{i=1}^n I_i = I_1 + I_2 + I_3 + \dots = 0$$

C. Devices

Capacitor

$$i(t) = C \left(\frac{dv}{dt} \right)$$

Inductor

$$v(t) = L \left(\frac{di}{dt} \right)$$

D. Modelling of Electrical Systems

	Voltage	Current
Resistors	$v(t) = i(t)R$	$i(t) = \frac{v(t)}{R}$
Capacitors	$v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau$	$i(t) = C \left[\frac{dv(t)}{dt} \right]$

Inductors	$v(t) = L \left[\frac{di(t)}{dt} \right]$	$i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau$
-----------	--	---

E. Trigonometric Identities

$\sin \theta = \pm \cos \left(\theta \mp \frac{\pi}{2} \right)$ $\sin \theta = -\sin(\theta \pm \pi)$ $\sin(-\theta) = -\sin \theta$	$\cos \theta = \pm \sin \left(\theta \pm \frac{\pi}{2} \right)$ $\cos \theta = -\cos(\theta \pm \pi)$ $\cos(-\theta) = \cos \theta$
$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$ $\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$	$\sin(2\theta) = 2 \sin \theta \cos \theta$
$2 \sin \theta \sin \phi = \cos(\theta - \phi) - \cos(\theta + \phi)$	$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$ $= 2 \cos^2(\theta) - 1$ $= 1 - 2 \sin^2(\theta)$

F. Time and Phasor Transformation

Time [x(t)]	Phasor [X]
$A \cos \omega t$	A
$A \cos(\omega t + \phi)$	$Ae^{j\phi}$
$-A \cos(\omega t + \phi)$	$Ae^{j(\phi \pm \pi)}$
$A \sin \omega t$	$Ae^{-j\pi/2} = -jA$
$A \sin(\omega t + \pi)$	$Ae^{j(\phi - \pi/2)}$
$-A \sin(\omega t + \pi)$	$Ae^{j(\phi + \pi/2)}$
$\frac{d}{dt}[x(t)]$	$j\omega \mathbf{X}$
$\frac{d}{dt}[A \cos(\omega t + \phi)]$	$j\omega Ae^{j\phi}$
$\int x(t) dt$	$\frac{1}{j\omega} \mathbf{X}$
$\int A \cos(\omega t + \phi) dt$	$\frac{1}{j\omega} Ae^{j\phi}$

G. Complex Number Relationship

Euler Identity:	
$e^{j\theta} = \cos \theta + j \sin \theta$	
Where: $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ and $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$	
Vector to Complex Number:	
$\mathbf{z} = x + jy = \mathbf{z} e^{j\theta}$ $x = \Re(\mathbf{z}) = \mathbf{z} \cos \theta$ $y = \Im(\mathbf{z}) = \mathbf{z} \sin \theta$ $\mathbf{z}^n = \mathbf{z} ^n e^{jn\theta}$	$\mathbf{z}^* = x - jy = \mathbf{z} e^{-j\theta}$ $ \mathbf{z} = \sqrt{\mathbf{z}\mathbf{z}^*} = \sqrt{x^2 + y^2}$ $\theta = \arctan(y/x)$ $\mathbf{z}^{1/2} = \pm \mathbf{z} ^{1/2} e^{j\theta/2}$
Operations of Vector:	
For $\mathbf{z}_1 = x_1 + jy_1$ and $\mathbf{z}_2 = x_2 + jy_2$	$\mathbf{z}_1 \mathbf{z}_2 = \mathbf{z}_1 \mathbf{z}_2 e^{j(\theta_1 + \theta_2)}$

$\mathbf{z}_1 = \mathbf{z}_2$ iff $x_1 = x_2$ and $y_1 = y_2$ $\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2)$	$\frac{\mathbf{z}_1}{\mathbf{z}_2} = \frac{ \mathbf{z}_1 }{ \mathbf{z}_2 } e^{j(\theta_1 - \theta_2)}$
Arithmetic of Complex Number:	
$-1 = e^{j\pi} = e^{-j\pi} = 1 \angle \pm 180^\circ$ $j = e^{j\pi/2} = 1 \angle 90^\circ$ $-j = e^{-j\pi/2} = 1 \angle -90^\circ$	$\sqrt{j} = \pm e^{j\pi/4} = \pm \frac{(1+j)}{\sqrt{2}}$ $\sqrt{-j} = \pm e^{j\pi/4} = \pm \frac{(1-j)}{\sqrt{2}}$

H. Op Amp

Inverting Amplifier

$$A_v = -\frac{R_f}{R_{in}}$$

Non-inverting Amplifier

$$A_v = 1 + \frac{R_2}{R_1}$$

Summing Amplifier

$$V_{out} = -\frac{R_f}{R_{in}} (V_1 + V_2 + V_3 + \dots)$$

Differentiator

$$V_{out} = -RC \left(\frac{dV_{in}}{dt} \right)$$

Integrator

$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

Difference Amplifier

$$V_{out} = \frac{R_4(R_1 + R_2)}{R_1(R_3 + R_4)} V_2 - \left(\frac{R_4}{R_3} \right) V_1$$

Total marks: 100 (Worth 24%)

Below are seven problems for this test. Do your calculations as needed and then put your answers (both working and final) as well any discussion or plots in a new document. Submit your answers with a filename: EEEN203_Test2_2024_” your surname”-“your initial” on the Wiki submission system no later than Friday 14 June.
Note: You are allowed to work in a team and share knowledge but let me know who you worked with so that I don’t penalise if the two solutions are similar.

Name:

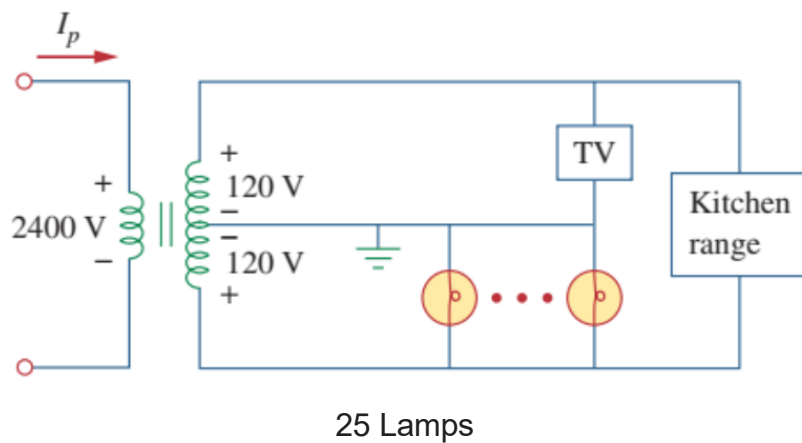
Student Number:.....

Q1. An ideal transformer is rated at 2400/220 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

[10 Marks]

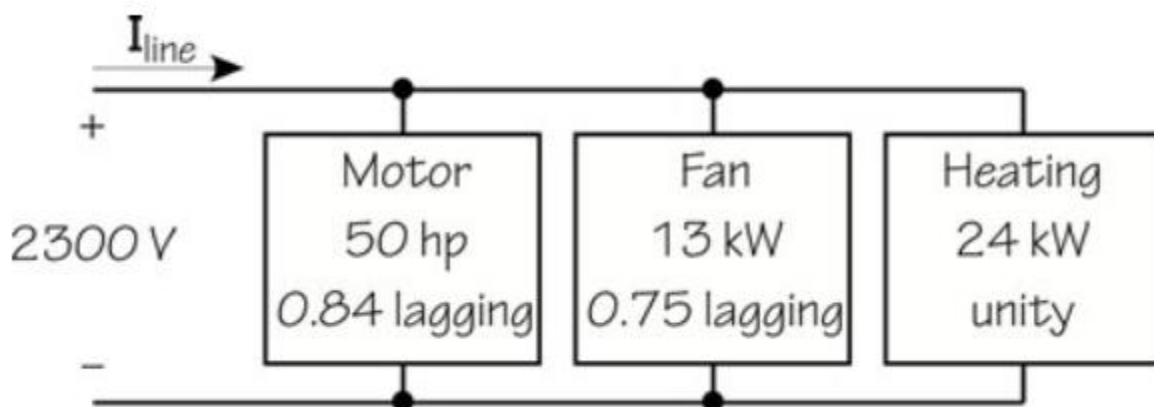
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[10 Marks]



Q3. Find Real, Reactive and Apparent Power for the following load configuration.

[20 Marks]



Q4. When connected to a 240-V (rms), 50-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.5. Find the value of capacitance necessary to raise the power factor to 0.95. **[15 Marks]**

Q5. A load Z draws 15 kVA at a power factor of 0.6 lagging from a 240-V rms sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance. **[15 Marks]**

Q6. A 1200-kW load supplied at 13 kV (rms) operates 410 hours a month at 76 percent power factor. Calculate the average cost per month based on this simplified tariff:

- Energy charge: 8 cents per kWh
- Power-factor penalty: 0.2 percent of energy charge for every 0.01 that pf falls below 0.85.
- Power-factor credit: 0.24 percent of energy charge for every 0.01 that pf exceeds 0.85.

[10 Marks]

Q7. Determine the total average, reactive, and complex powers at the source and at the load for the following figure. **[20 Marks]**

